Copyright

by

Romana Louise Autrey

2005

The Dissertation Committee for Romana Louise Autrey certifies that this is the approved version of the following dissertation:

Three Essays on Teams and Synergy

Committee:

D. Paul Newman, Supervisor

Shane S. Dikolli, Co-Supervisor

R. Preston McAfee

Urton L. Anderson

Stephen M. Gilbert

Three Essays on Teams and Synergy

by

Romana Louise Autrey, B.S.

Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

August 2005

Dedication

This dissertation is dedicated to my husband, Tito, and to my best friend and inspiration, Erin Smyth, for their unwavering support.

Acknowledgements

I would like to thank the members of my committee for their direction and guidance: Paul Newman (co-chairman), Shane Dikolli (co-chairman), Urton Anderson, Preston McAfee and Steve Gilbert. For their comments, I would also like to thank Tito Autrey, Jessen Hobson, Steve Kachelmeier, Ana Marques, Karen Sedatole and Peggy Weber and the workshop participants at the University of Texas at Austin, University of Washington, INSEAD, University of Illinois at Urbana-Champaign, University of Utah, Northwestern University, and Harvard University, and the conference participants at the Fourth Accounting Research Workshop by the Universities of Bern & Fribourg. I gratefully acknowledge the financial support of the Deloitte Foundation.

3 August 2005

Three Essays on Teams and Synergy

Publication No. _________

Romana Louise Autrey, Ph.D.

The University of Texas at Austin, 2005

Supervisor: Donald Paul Newman; Co-Supervisor: Shane S. Dikolli

The pursuit of synergy, where the whole exceeds the sum of the parts, inspires the formation of teams: working together, team members can create synergy, and hence value, for a firm. This dissertation explores the interaction of synergy and team member characteristics under various performance measurement regimes. Specifically, I analytically model the impact of this interaction on the explicit and implicit incentives facing each team member and the resulting types and amounts of effort that team members choose. The results indicate that team composition and synergy play an important role in determining which performance measurement regime generates the highest agency welfare. A high-synergy setting favors the inclusion of a team output measure that encompasses this synergy, whereas a lower-synergy setting may favor individual input measures that do not reflect this synergy. Accordingly, no one performance measurement regime dominates in all circumstances.

Table of Contents

List of Figures ix

Chapter 1: Introduction 1

Chapter 2: Team Synergy, Team Composition and Performance Measures 4

2.1. Introduction 4

2.2. Prior Research 8

2.3. The Model 10

2.4. Analysis 14

2.4.1 Benchmark: First-Best effort 14

2.4.2 Individual performance measures contractible 16

2.4.2.1 Regime Y: Contracting only on a measure of individual effort 16

2.4.2.2 Regime Z: Contracting only on a measure of cooperative effort 17

2.4.2.3 Regime YZ: Contracting on both individual measures 19

2.4.3 Regime X: Team output contractible 20

2.4.4 Regime XY: Mix of team and individual performance measures contractible 24

2.5. Conclusion 28

Chapter 3: Differential Career Concerns and Team Synergy 30

3.1. Introduction 30

3.2. The Model 35

3.3. Analysis 37

3.3.1 Team output measure observable (Regime *X*) 39

3.3.2 Individual performance measures observable (Regime *Y*) 47

3.3.3 All performance measures observable (Regime *XY*) 51

3.3.4 Comparison across regimes 59

3.4. Conclusion 61

Chapter 4: Team Synergy, Career Concerns and Incentive Pay 64
4.1. Introduction 64
4.2. The Model 65
4.3. Analysis 68
4.3.1 Team output measure contractible (Regime *XC*) 69
4.3.2 Individual performance measures contractible (Regime *YC*) 72
4.3.3 All performance measures contractible (Regime *XYC*) 77
4.3.4 Comparison across regimes 80
4.4. Comparison Across Contractible and Non-Contractible Regimes 82
4.5. Conclusion 83

Appendix A 85

Appendix B 91

Appendix C 95

Glossary 102

References 103

Vita 106

List of Figures

Figure 2.1. Firm profit across regimes as a function of σ_x and Σ_y (p=q=r=1, s=2, Σ_y^2=σ_x^2, Σ_z^2=1) and as a function of Σ_z (p=q=r=1, s=2, Σ_y^2=3/4, σ_x^2 =1). 26

Figure 2.2. Firm profit as a function of task expertise p (q=1,r=1, s=2, σ_x^2 =5, Σ_y^2=Σ_z^2=1) and as a function of team expertise q (p=2,r=1, s=2, σ_x^2 =.5, Σ_y^2=Σ_z^2=1). 27

Figure 3.1. Expected firm profit in the X regime as a function of k_i. (m=1, r=.5, σ^2=.25, s=4, k_j=1)...44

Figure 3.2. Cross-partial derivative on profit in the X regime with respect to career concerns. (m=1, r=.5, σ^2=.25, s=2, left panel k_j=1, right panel k_j=2)...45

Figure 3.3. Expected firm profit in the Y regime. (f_{ii}=.25, f_{jj}=.25, k_j=1, n=1, r=.5, σ^2=.25, s=2) 48

Figure 3.4. Expected firm profit in the Y regime. (f_{11}= f_{22} =.25, n=1, r=.5, σ^2=.25, s=1/2) 50

Figure 3.5. Cross-partial derivative on teamwork in the XY regime with respect to each agent's career concerns. (k_j=10, m=1, n=10, s=2) 55

Figure 3.6. Expected firm profit in the XY regime. (f_{11}=f_{22}=0,s=2,k_1=3,k_2=1, m=3, r=.5, σ^2=.25) 56

Figure 3.7. Expected firm profit in the XY regime. (f_{11}=f_{22}=0, k_j=.1,m=.25,n=4, r=.5, σ^2=.25, s=2)...56

Figure 3.8. Expected firm profit in the XY regime. (f_{11}=f_{22}=0, k_j=1, m=1, n=4, r=.5, σ^2=.25, s=2)...57

Figure 3.9. Expected firm profit in the *X*, *Y* and *XY* regimes. (f_{11}= f_{22} =0, m=1, n=1, r=.5, σ^2=.25, left panel: s=2, right panel: s=10) 60

Figure 3.10. Expected firm profit in the *X* and *Y* regimes. (f_{11}= f_{22} =0, m=1.5, n=1.5, r=.5, σ^2=.25,s=1) 61

Figure 4.1. Rate of change of effort in career concerns in the *YC* regime (f_{ii}=1, n=1, r=1,σ^2=1, left panel s=9, right panel s=0.5) 75

Figure 4.2. Impact of interaction between synergy and career concerns on effort, in *XYC* regime (f_{ii}= f_{ii}=0, k=1, m=1, n=4 r=σ^2=1, s=1)...........................79

Figure 4.3. Expected firm profit in the *XC* and *YC* regimes (k=3, m=1.5, n=1.5, r=2,σ^2=1, left panel f_{ii}= f_{ii}=0, right panel f_{ii}= f_{ii}=1).................................81

Chapter 1: Introduction

The American Heritage Dictionary defines synergy as "the interaction of two or more agents or forces so that their combined effect is greater than the sum of their individual effects." The pursuit of synergy inspires the formation of teams: working together, team members can create synergy, and hence value, for a firm. This dissertation explores the interaction of synergy and team member characteristics under various performance measurement regimes. Specifically, I study the impact of this interaction on the explicit and implicit incentives facing each team member and the resulting types and amounts of effort that team members choose.

This is an important research topic because of the extensive economic impact of teams in business: every firm is essentially a team attempting to generate synergy. Not only is there a lot of money at stake, but creating synergy is difficult to do. In practice, different experts make conflicting recommendations to firms seeking to induce teamwork and reap the benefits of synergy. This analysis attempts to reconcile this contradictory guidance and clarify the role of each of several forces contributing to synergy.

This dissertation presents three analytic models of teams and synergy. The first model develops a one-period benchmark in which team members have different cost advantages, or "expertise." The second model builds on the one-period base model by adding a second period; here, team members have different levels of career concerns and no performance measures are contractible. The third model extends this two-period setting to include contractible performance measures.

In all three models, two team members each choose two types of effort: the first, teamwork, increases their teammate's marginal productivity and the second, individual effort, has no impact on their teammate's marginal productivity. Synergy is modeled as the interaction between agents produced by teamwork. All models vary the performance measurement regime by manipulating the performance measures available for observation and/or contracting. Performance measurement regimes considered include a regime where only a team output measure is available, a regime where only individual input measures are available, and a regime where a mix of team output and individual inputs is available.

The results indicate that when team members influence each other's marginal productivity via teamwork, each agent's teamwork choice depends not only on his own incentive weights and characteristics, but also on the incentive weights and characteristics of his teammate. Further, I find that no one performance measurement regime dominates in all circumstances.

In a one-period model, ceteris paribus, higher potential synergy favors team output regimes whereas a higher ratio of team-to-task expertise favors individual input regimes. In a two-period model with non-contractible performance measures, the potential for synergy interacts with career concerns to produce collaborative effort. In a high synergy environment, firms are better off having at least one performance measure sensitive to potential synergy, or team members under-invest in teamwork. Conversely, in a low synergy environment, firms are better off having at least one performance measure that is *not* sensitive to potential synergy, or team members over-invest in teamwork. Further, even in the absence of explicit costs for each performance measure, more performance measures are not necessarily better.

In a two-period model with contractible performance measures, firms are better off with higher synergy potential and teams of agents with low career concerns ("senior agents"). In general, synergy reinforces the benefit from senior agents. This contrasts with non-contractible performance measurement regimes, where firms are better off with teams of agents with high career concerns ("junior agents"), and synergy causes those junior agents to be even more valuable to the firm. The reason for this difference is that in the absence of explicit incentives, a firm must rely on career concerns to induce effort (so the more career concerns, the more effort), whereas when a firm can induce effort via explicit incentives, career concerns distort the effort level away from the second best level (and the more career concerns, the more distortion).

Chapter 2: Team Synergy, Team Composition and Performance Measures

2.1. INTRODUCTION

Team synergy occurs when a team's output exceeds the sum of the output of the team members working individually.[1] However, synergy requires more than merely assigning individuals to a team (Lawford [2003], Salas, Burke, & Cannon-Bowers [2000]). It is enabled by a collaborative environment, which is created by a firm establishing and then maintaining a corporate culture that nurtures and reinforces teamwork.[2] As part of this process, many firms employ explicit team-based incentives (Parker, McAdams, & Zielinski [2000]). In practice, guidance to firms on the appropriate performance measures for inducing teamwork differs. For example, Lawford [2003] recommends that firms *not* single out individual team members for special acknowledgement, to prevent competitiveness that can damage a collaborative environment. On the other hand, Parker *et al.* [2000] advise firms to "send the right message" by reinforcing teamwork *and* individual performance. This chapter attempts to reconcile this contradictory advice by examining several factors that influence a firm's effectiveness at inducing teamwork to reap the benefits of synergy, under various performance measurement regimes.

The level of teamwork a firm can induce using incentives ultimately depends on the type of performance measures available to the firm. In an ideal world, performance measurement in a team-based setting is straightforward: a firm perfectly observes both

---

[1] Synergy is also cited as the justification for strategic alliances and company mergers/acquisitions. I do not consider this scenario. Rather, I focus on operational synergy within a firm.

[2] The importance of a collaborative environment is evidenced by PeopleSoft's firing of its CEO, Craig Conway, citing concerns that its collaborative work style was in danger (Clark [2004]).

the individual contributions of each team member and the total value generated by a team. However, in reality, synergy creates a number of measurement problems that complicate the use of incentive plans.

One fundamental problem is measuring how much value each individual team member provides. Alchian and Demsetz [1972] assert that in a team setting it is prohibitively costly to ascertain the marginal product of each team member. But, even if such practicalities are overcome and marginal products of each team member are available, Rose [2002] demonstrates the fundamental impossibility of paying each team member his marginal product because synergy causes the sum of the individual marginal products to exceed the collective marginal product. Furthermore, there is no reasonable way to allocate team synergy to individual team members (Watts [2003]). This allocation problem is similar to the classic managerial accounting "joint cost/benefit" problem.

Failing the ability to measure and/or pay team members based on their marginal products, a firm may attempt to measure, as a proxy, effort inputs by team members. For example, one measure of a team member's individual effort is the time each team member devotes to production. An example of a measure of a team member's cooperative effort, or teamwork, is the time each team member invests in coordinating with other team members to improve performance. However, team settings are notoriously difficult for disentangling the effort contributions of individual team members as a result of the very synergy for which the team was formed (McAfee and McMillan [1991]).[3] For example, attributing the total revenue generated by an

[3] A related problem, free-riding, is invited by the inability of firms to measure individual effort and/or marginal products of individual team members. Free-riding, also known as social loafing or shirking, occurs when team members do not internalize the full positive impact of their actions on their teammates. Thus, each individual provides less effort than is optimal for the team as a whole (as with the under-provision of public goods) because the marginal benefit to the team is higher than the marginal benefit to the individual.

interdepartmental sales team to individual departments without double- and triple-counting is often problematic.

Another measurement problem in team settings is that sometimes a firm cannot quantify the benefits generated by a team, which implies that team output may not be available for contracting. Firms find it difficult to put a dollar value on improvements in quality, productivity, customer satisfaction, employee turnover and cycle time. Despite this inability to measure results, firms still perceive that team-based incentive plans are good for the organization (Parker *et al.* [2000]). For example, even when a firm cannot directly measure the benefits of a major team-led IT implementation, the firm may still believe that the resulting information system provides substantial value.

The research issue addressed in this chapter is important to firms because of the potentially high stakes involved and the difficulty involved in designing team-based incentives. Che and Yoo [2001] cite numerous examples of successful teams that improved profits – generating additional revenue, increasing productivity, reducing engineering delays, and decreasing cycle times – by as much as $50 million at a single firm. Porter [1996] claims that a firm's very competitiveness is fundamentally driven by the synergistic fit among workers and the tasks they perform. This question is also important to accounting researchers because it addresses the impact of performance measurement limitations on the ability of firms to generate synergy. I manipulate the information environment a firm faces with respect to the availability of performance measures used to induce teamwork among its employees and analyze which measurement regimes perform better under which circumstances.

In my model, workers perform two types of effort, individual effort and cooperative effort, with associated cost efficiencies (respectively, task expertise and team expertise). This dichotomy is consistent with the categorization in the team literature of team member competencies into task and team skills (Salas *et al.* [2000]). Individual effort involves tasks that can be performed non-collaboratively (*e.g.*, individual sales), whereas teamwork entails tasks which require coordination among workers (*e.g.*, after-sales service, involving communication across several departments). The extant principal-agent literature considers the former type of effort extensively. I focus on the latter and its role in generating synergy, which has received relatively little attention by modelers. I emphasize teamwork because it is the mechanism by which team members interrelate, adapting and adjusting the timing of their individual actions to meet the demands placed upon the team. Empirical evidence suggests that this coordination is essential to effective team performance (Salas *et al.* [2000]).

This chapter presents a LEN model with two agents who each choose both individual effort and cooperative effort (*i.e.*, teamwork).[4] Both types of effort contribute to team output; however, for clarity, I refer to the effort with a marginal impact on teammates as "teamwork" and the effort with no marginal impact as individual effort.[5] I model synergy as an increasing, concave function of teamwork, which takes a value of zero if either team member chooses no teamwork. Team output, which may or may not be contractible, includes this synergy component, whereas the individual performance measures of teamwork and individual effort inputs do not.

I find that when team members influence each other's marginal productivity via teamwork, each agent's teamwork choice depends not only on his own incentive weights and team expertise (*i.e.*, cooperative effort cost advantage), but also on the incentive weights and team expertise of his teammate. Further, I find that firm profit can be higher in either a team-output-based or a teamwork-input-based contracting regime. An increase

---

[4] A LEN model assumes wage contracts that are Linear functions of the performance measures, agents with negative Exponential utility functions, and Normally distributed performance measures.

[5] I use the terms "teamwork" and "cooperative effort" interchangeably.

in team expertise is likely to benefit a firm with a *task*-focused team (*i.e.*, relatively more task expertise than team expertise) most in a team-*output*-based contracting regime, whereas a firm with a *team*-focused team is likely to benefit most in a teamwork-*input*-based contracting regime. Finally, firm profit can be higher when contracting only on two types of input-based individual performance measures than when contracting on a blend of team output and one type of input measure. Presuming the firm has a limited budget for implementing its performance measurement system and cannot afford to produce all measures, the most valuable measure(s) to the firm for contracting purposes depends not only on the precision of the measures, but also on team composition and the degree of synergy.

The rest of this chapter is organized as follows. The next section reviews prior research. The following section describes the model setup. In the analysis section I manipulate the information environment and analyze the differences across performance measurement regimes. The final section concludes and discusses limitations.

2.2. PRIOR RESEARCH

This chapter builds on the existing analytic team and performance measurement literatures. The relevant team literature contains several discrete models and a small number of continuous models with a team synergy component; these models generally hold the performance measurement environment constant. The performance measurement literature generally models the impact of performance measure characteristics on incentive compensation contracts (*e.g.*, Feltham and Xie [1994], Dikolli [2001], Datar, Kulp, & Lambert [2001]). My model bridges both literatures by examining the role of performance measure characteristics in the relatively unexplored team synergy setting.[6] Specifically, this chapter contributes to the team literature by comparing the impact of different information regimes on effort choices and overall team performance in a team synergy setting. It contributes to the performance measurement literature by comparing the impact of team composition on effort choices and overall team performance in a team synergy setting.

The team literature has many discrete models that incorporate a synergy-like component. For example, Arya, Fellingham, & Glover [1997] and Che and Yoo [2001] model two identical agents who make binary effort choices, where the output from cooperative (high, high) effort can exceed the sum of the agents choosing high effort in isolation (high, low)[7]. However, these papers assume a specific performance measure regime – a team performance measure only in Arya *et al.* [1997] and individual performance measures only in Che and Yoo [2001] – and focus on the role of the *agent* in inducing teamwork. In contrast, I vary the availability of performance measures of continuous effort choices and focus on the *principal's* role in inducing teamwork.

Continuous models of teams with a synergy component are rare. McAfee and McMillan [1991] model synergy in a team setting with *n* risk-neutral agents who choose a vector of continuous effort inputs. Although primarily an adverse selection model, they consider the special case where adverse selection is absent. Their main finding is that when the marginal cost/benefit ratio is non-decreasing in ability for all agents, the first-best outcome can be attained using incentive contracts linear in team output alone.[8] While McAfee and McMillan [1991] do consider the incremental value of adding

[6] The research stream concerning task assignment/ task complementarities has a similar flavor in that it entails multiple agents and complementary tasks (*e.g.*, Zhang [2003], Hemmer [1995]). These papers examine the interaction of complementarity and job design/ organizational design (as opposed to performance measure characteristics), and they model exogenous complementarity across *tasks* rather than endogenous complementarity across *agents* (i.e., synergy).

[7] Similar examples include Macho-Stadler and Perez-Castrillo [1993] and Itoh [1993].

[8] That is, monitoring is unnecessary to eliminate free-riding for risk-neutral agents. Vanderveen [1995] extends this model to risk-averse agents and finds that monitoring does prevent some free-riding.

noiseless individual performance measures, the team-output-only regime weakly dominates any other performance measurement regime because the first-best solution can be attained.

Holmstrom [1982] investigates moral hazard in teams of *n* agents who each make a single continuous effort choice. However, he considers scenarios where synergy is completely allocated to individual agents or is not present. In my model, synergy (by definition) causes the team output to exceed the sum of individual performance measures (*i.e.*, it cannot be allocated to individual agents). This assumption removes the "budget-balancing constraint" that underlies Holmstrom's model. Auriol, Friebel, & Pechlivanos [2002] develop a LEN model of two agents who each choose a single continuous effort that has a positive externality on the teammate. As with Holmstrom [1982], team output is the sum of the individual agent outputs. The externality, however, does not represent synergy per se because agents have no effect on their teammates' marginal product.

This chapter is most closely related to the Auriol *et al.* [2002] and Datar *et al.* [2001] models. I extend the Auriol *et al.* [2002] model to include a nonlinear team synergy term in team output, and I introduce individual performance measures that do not capture this synergy. Thus, each team member's teamwork has a marginal impact on the other's productivity in team output that is not reflected in the individual measures. I extend the Datar *et al.* [2001] single-agent multiple-action model to a multiple-agent multiple-action model where each agent has different characteristics.

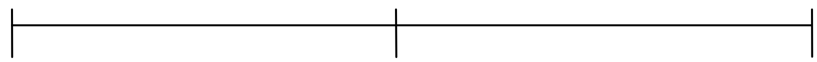
2.3. THE MODEL

Consider a one-period model where a risk-neutral principal hires two risk- and effort-averse agents. Each agent chooses individual effort, e_i, and teamwork, τ_i.[9]

[9] Synergy could be represented with a single type of effort. However, I use two types of effort because I want to model teamwork as a separate construct from individual effort. I would like to highlight the contrast between effort that contributes to output additively (as in the prior literature) versus

Individual effort is reflected in both team output, x, and an individual performance measure, y_i, if one exists. Likewise, teamwork is reflected in both team output, x, and an individual performance measure, z_i, if one exists. Synergy is a function of teamwork by the two agents: teamwork by one agent increases the other agent's marginal productivity.

The timeline is as follows:



Team output is a linearly additive function of individual efforts e_1 and e_2, a team synergy term, $s(\tau_1\tau_2)^{1/2}$, and a transient shock, ε_x , as follows:[10]

$$x = e_1 + e_2 + s(\tau_1\tau_2)^{1/2} + \varepsilon_x$$

where s>0 is an exogenous parameter known to all parties that represents the strength of the synergy.[11] Depending on the measurement regime, x may or may not be contractible.

The individual performance measures, if they exist, are linearly additive functions of individual effort e_i and teamwork τ_i and a transient shock, ε_{yi} and ε_{zi}, respectively:

$$\left.\begin{aligned} y_i &= e_i + \varepsilon_{yi} \\ z_i &= \tau_i + \varepsilon_{zi} \end{aligned}\right\} \quad i = 1,2$$

For example, Parker *et al.* [2000] document that Ameritech internal audit services evaluates the time spent developing new audit methodologies (measure y) and the time spent sharing best practices with others (measure z) for its incentive plan. In some cases,

multiplicatively (which creates the marginal interdependency between agents). The combination of these two types of effort has the intuitive feature that team output when agents collaborate exceeds the sum of individual agent output when agents work individually (i.e., no teamwork).

[10] I model the synergy term as the square root of $\tau_1\tau_2$ to ensure an interior solution with a quadratic cost function for τ. I find this assumption palatable because it implies synergy is increasing at a decreasing rate in teamwork.

[11] Parameter s is analogous to the sensitivity of team output to teamwork, as modeled in Datar *et al.* [2001]. Modeling this parameter as a random variable is not feasible in the LEN setup because multiplying a choice variable (in this case, τ) by the random variable causes the agent's choice to change the variance of the performance measure, which creates an insurmountable tractability problem (Lambert [2001]).

however, these individual performance measures might not be contractible. Chase Manhattan found that its local performance measurement systems were incompatible with each other, which meant the desired data could not be collected. This issue is common enough that one of the main takeaways in Parker *et al.* [2000] is for firms to assess the contractibility (*i.e.*, availability and reliability) of performance measures when designing the plan. Alternatively, meaningful measures of unobservable effort simply may not exist. Consider a co-authored paper: when asked to self-report their percentage of the contribution, authors almost invariably split credit evenly with their co-authors.

All transient shock terms are normally distributed with zero mean and variances σ_x^2, σ_{y1}^2, σ_{y2}^2, σ_{z1}^2, and σ_{z2}^2 respectively, where the σ^2 parameters are all positive and finite. The variance terms represent the precision of each performance measure (*i.e.*, higher values of σ^2 imply less precise measures). For example, a creative team might have performance measures with low precision, whereas a production team might have performance measures with high precision.

The individual performance measures have corr(y_1, y_2)=ρ_y, corr(z_1, z_2)=ρ_z; for simplicity, I restrict these correlations to be non-negative (ρ_y, $\rho_z \geq 0$). The individual performance measures are independent, *i.e.*, corr(y_i, z_i)=0. The team output shock term, ε_x, is independent of the shock term in both individual performance measures. One interpretation of the correlation ρ_y between the ε_{yi} shock terms is the degree of similarity between team members: a cross-functional team could have quite low correlations, whereas a specialized vertical team could have high correlations. Similarly, the correlation ρ_z between the ε_{zi} shock terms might represent the extent to which teamwork is measured at the team level versus measured separately by each home department. The performance measures are normally distributed as follows:

$$x \sim N(e_1 + e_2 + s(\tau_1\tau_2)^{1/2}, \sigma_x{}^2)$$
$$\left.\begin{array}{l} y_i \sim N(e_i, \sigma_{yi}{}^2) \\ z_i \sim N(\tau_i, \sigma_{zi}{}^2) \end{array}\right\} \quad i = 1,2$$

The principal offers each agent a wage based on the measures available for contracting. For tractability, I assume contracts take the linear form:

$$w_i = \alpha_i + \beta_i x + \gamma_i y_i + \delta_i y_j + \kappa_i z_i + \lambda_i z_j, \quad \text{for } i, j = 1,2, \ i \neq j$$

where the incentive weight on any non-contractible measure equals zero.

Each agent's cost of effort is a twice-differentiable convex increasing function of individual effort and teamwork. For simplicity, assume the costs of each type of effort are additively separable. The cost to agent *i* for individual effort is decreasing in his task expertise, $p_i \geq 1$, and the cost of teamwork is decreasing in his team expertise, $q_i \geq 1$.[12] Total cost of effort has the following functional form:

$$c_i = \frac{1}{2p_i} e_i^2 + \frac{1}{2q_i} \tau_i^2, \quad \text{for } i = 1,2 \tag{1}$$

Each agent has a constant absolute risk aversion (CARA) utility function:

$$U_i \equiv -\exp\{-r_i[w_i - c_i]\}, \quad i = 1,2$$

where $r_i \in (0,\infty)$ is the Arrow-Pratt measure of absolute risk aversion for agent *i*. For simplicity, and without loss of generality, assume each agent's outside reservation utility equals zero.[13] Task expertise, team expertise and risk aversion (p_i, q_i and r_i) collectively represent team composition, or the characteristics of the team's members.

The principal is risk neutral and maximizes expected profit:

---

[12] Although outside the scope of this model, one might expect team members to possess skills appropriate to their working environment. For example, high team expertise would be useful in project management, but task expertise is less useful. Writing software code requires high task expertise but not necessarily team expertise. A lead attorney in a complex trial requires both high task expertise *and* high team expertise, whereas a retail clerk might not need high expertise of either type.

[13] Alternatively, each agent's reservation utility could be a function of his individual characteristics, which are known to all parties. However, in a static model, the principal will merely adjust the fixed wage α_i to ensure the outside reservation utility is met, and the slope of the pay-for-performance will be unchanged.

$$\left.\begin{aligned} &\underset{\alpha_i,\beta_i,\gamma_i,\delta_i,\kappa_i,\lambda_i}{\text{Max}} \quad \Pi \equiv Ex - Ew_1 - Ew_2 \\ &\text{subject to (PC) } EU_i \geq 0 \\ &\qquad\qquad\quad \text{(IC) } e_i, \tau_i \in \arg\max EU_i^{\,N} \end{aligned}\right\} \; i = 1,2 \tag{2}$$

where $N \in \{X, Y, Z, XY, YZ\}$ refers to the regime where only performance measures $n \in \{x, y, z, (x,y), (y,z)\}$ respectively are available for contracting. Here, (PC) represents the participation constraint and (IC) is the incentive compatibility constraint.

2.4. ANALYSIS

In this section, I manipulate the information environment a firm faces. The performance measures that a firm has available for contracting depend on the nature of the team project and the types of activities team members undertake. Some activities and projects are inherently less measurable than others. For example, straightforward production tasks are more measurable than tasks involving creativity; an incremental cost-reduction project is more measurable than complex and/or interrelated projects such as new product development. I compare the difference in firm profit across various measurement regimes to evaluate the relative advantage of each regime. In other words, I compute the relative value of performance measures to the firm, *ceteris paribus*.

First, I develop a first-best effort benchmark for comparison with other regimes. Second, I consider regimes where various combinations of individual performance measures are contractible, but team output is not. Third, I present the regime where only team output, *x*, is available for contracting. Finally, I present the regime where a mix of team output and individual performance measures is available.

2.4.1 Benchmark: First-Best effort

In the first-best regime, the principal observes the level of individual effort and teamwork that each agent chooses and pays the agent a fixed wage that covers the agent's effort cost, if (and only if) the agent has exerted the desired level of each type of effort. To determine the first-best level of effort, the principal maximizes the following:

$$\mathrm{E}[x] - \mathrm{E}[w_i] - \mathrm{E}[w_j] = e_i + e_j + s(\tau_i \tau_j)^{1/2} - c_i - c_j \tag{3}$$

To solve, substitute equation (1) into the above and take first order conditions with respect to efforts e_i and τ_i, which yields:

$$e_i^{FB} = p_i, \ \tau_i^{FB} = \frac{4}{s^2 q_j^2}\left(\tau_j^{FB}\right)^3 \quad \text{for } i, j = 1, 2, \ i \neq j$$

where the superscript *FB* refers to the first-best regime. In the first-best regime, each agent's individual effort is a simple increasing function of his task expertise. Note that each agent's choice of teamwork is an increasing, convex function of his *teammate's* teamwork. Solving for both agents simultaneously yields the optimal teamwork levels. Substituting the first-best effort levels into (3) yields firm profit. In general, maximum profit in regime *N* is of the form:

$$\Pi^N = \frac{1}{2}(e_i^N + e_j^N) + \frac{s}{2}(\tau_i^N \tau_j^N)^{1/2} \tag{4}$$

Firm profit consists of two terms: the arithmetic mean of individual effort, and the geometric mean of teamwork times half the synergy parameter. The first-best effort levels and firm profit are as follows:

$$\begin{aligned} &e_i^{FB} = p_i, \quad \tau_i^{FB} = \frac{s}{2} q_i^{3/4} q_j^{1/4}, \quad \text{for } i, j = 1, 2, \quad i \neq j \\ &\Pi^{FB} = \frac{1}{2}(p_i + p_j) + \frac{s^2}{4}\left(q_i q_j\right)^{1/2} \end{aligned} \tag{5}$$

Team composition plays a role in each agent's optimal teamwork choice even in the first-best regime: agent *i*'s teamwork increases not only in his own team expertise q_i, but also in his teammate's team expertise q_j. Firm profit is easily verified using equation (4) for the first-best regime.[14] In this model, the first-best is not attainable due to risk-averse agents and imperfect precision.

2.4.2 Individual performance measures contractible

In this subsection I model the following regimes: (i) only the individual performance measure of individual effort, *y*, is available for contracting (regime *Y*), (ii) only the individual performance measure of teamwork, *z*, is available for contracting (regime *Z*), and (iii) both individual performance measures are available for contracting (regime *YZ*). Regimes *Y* and *Z* are provided primarily for completeness and to facilitate determining the relative value to a firm of adding a performance measure. In regimes *Y* and *Z*, I assume that only one of the individual measures is available for contracting and that the other individual measure and team output are prohibitively costly to obtain.

2.4.2.1 Regime Y: Contracting only on a measure of individual effort

Consider a regime where only performance measure *y* is contractible. The principal offers each agent a wage of the form $w_i = \alpha_i + \gamma_i y_i + \delta_i y_j$. Agent *i*'s problem is to maximize his certainty equivalent as follows:[15]

$$\underset{e_i,\tau_i}{\text{Max}}\ \mathrm{E}w_i - c_i - \frac{r_i}{2}\operatorname{var} w_i = \alpha_i + \gamma_i e_i + \delta_i e_j - \frac{e_i^2}{2p_i} - \frac{\tau_i^2}{2q_i} - \frac{r_i}{2}(\gamma_i^2\sigma_{yi}^2 + \delta_i^2\sigma_{yj}^2 + 2\gamma_i\delta_i\rho_y\sigma_{yi}\sigma_{yj})$$

Taking first order conditions with respect to efforts e_i and τ_i, and solving for the optimal effort levels yields $e_i^Y = p_i\,\gamma_i^Y$ and $\tau_i^Y = 0$. In this regime, no teamwork is induced because the principal has no access to a performance measure sensitive to teamwork. Substituting the (PC), (IC) constraints into (2) yields the principal's objective function:

$$\Pi^Y = \sum_{i=1,2}\left(p_i\gamma_i - c_i - \frac{r_i}{2}\operatorname{var} w_i\right) \tag{6}$$

[14] Straightforward algebra demonstrates that firm profit in the first-best regime strictly dominates firm profit in all other regimes analyzed (i.e., $\Pi^{FB} > \Pi^N$ for all $N \in \{X, Y, Z, XY, YZ\}$).

[15] Given a linear wage, the agent's exponential utility function, and the normality assumptions on y_i, the agent's expected utility EU_i can be written in terms of the agent's certainty equivalent, as presented.

To solve for the optimal incentive weights in regime *Y*, take first order conditions with respect to each incentive weight and solve. Substituting these optimal incentive weights into equation (6) yields optimal firm profit for regime *Y*.

$$\gamma_i^Y = \frac{p_i}{p_i + r_i \sigma_{yi}^{\ 2}(1-\rho_y^{\ 2})}, \quad \delta_i^Y = -\gamma_i^Y \rho_y \frac{\sigma_{yi}}{\sigma_{yj}}$$
$$\Pi^Y = \frac{1}{2}\left(\frac{p_i}{p_i + r_i \sigma_{yi}^{\ 2}(1-\rho_y^{\ 2})} + \frac{p_j}{p_j + r_j \sigma_{yj}^{\ 2}(1-\rho_y^{\ 2})} \right) \tag{7}$$

In this regime, the incentive weight offered to agent *i* on his performance measure is an increasing function of his task expertise p_i. It is also a simple decreasing function of his risk aversion and the precision of his personal performance measure, a standard result in LEN models. The relative performance evaluation (*i.e.*, incentive weight δ_i^Y) offered to each agent on his teammate's performance measure is negative, which represents the filtering out of common variance from an agent's compensation. Note that when the team members' individual performance measures are not correlated (*i.e.*, ρ_y=0), there is no relative performance evaluation (*i.e.*, δ_i^Y=0), another standard result in the literature. The resulting profit is a simple average of each agent's individual effort.

2.4.2.2 Regime Z: Contracting only on a measure of cooperative effort

Next, consider a regime where only performance measure *z* is contractible. The principal offers each agent a wage of the form $w_i = \alpha_i + \kappa_i z_i + \lambda_i z_j$. Agent *i*'s problem parallels the *Y* regime where only a measure of individual effort is available. Taking first order conditions of the agent's certainty equivalent with respect to efforts e_i and τ_i and solving for the optimal effort levels yields $e_i^Z = 0$ and $\tau_i^Z = q_i \kappa_i^Z$.[16] In this regime, no

[16] There is no subgame problem in this or any regime modeled. *I.e.*, neither agent has a profitable deviation when his teammate chooses the equilibrium effort level. Furthermore, although some of the regimes have multiple equilibria, there is only one equilibrium in each regime with positive effort levels: the other equilibria have zero, negative or complex effort levels.

individual effort is induced because the principal has no access to an appropriately sensitive performance measure. The principal maximizes:

$$\Pi^Z = \frac{s}{2}(q_1\kappa_1 q_2\kappa_2)^{1/2} - \sum_{i=1,2}\left(c_i + \frac{r_i}{2}\operatorname{var} w_i\right) \tag{8}$$

To solve for the optimal incentive weights, take first order conditions with respect to each incentive weight and solve. Substituting these optimal incentive weights into equation (8) yields optimal firm profit for regime *Z*, as summarized in Lemma 2.1.

LEMMA 2.1. In regime *Z*, the optimal incentive weights and firm profit are:[17]

$$\begin{aligned}
\kappa_i^Z &= \frac{s\left(q_i q_j\right)^{1/2}}{2(q_i + r_i\sigma_{zi}^2(1-\rho_z^2))^{3/4}(q_j + r_j\sigma_{zj}^2(1-\rho_z^2))^{1/4}}, \quad \lambda_i^Z = -\kappa_i^Z \rho_z \frac{\sigma_{zi}}{\sigma_{zj}} \\
\Pi^Z &= \frac{s^2 q_i q_j}{4(q_i + r_i\sigma_{zi}^2(1-\rho_z^2))^{1/2}(q_j + r_j\sigma_{zj}^2(1-\rho_z^2))^{1/2}}
\end{aligned} \tag{9}$$

In contrast to the *Y* regime, the incentive weight the principal offers to each agent in the *Z* regime is a function of *both* agents' team expertise, risk aversion and individual measure precision. This property has implications for a cross-functional team, which is generally comprised of team members from diverse departments, with potentially widely varying characteristics (especially measurement precision) due to the diversity of activities. The model predicts that the optimal level of teamwork for each team member varies based on team composition.

Comparing firm profit under the *Y* and *Z* regimes yields the following:

$$\begin{aligned}
&\Pi^Y > \Pi^Z \Leftrightarrow \frac{2D_Z}{q_i q_j}\left(\frac{p_i}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)} + \frac{p_j}{p_j + r_j\sigma_{yj}^2(1-\rho_y^2)}\right) > s^2 \\
&\text{where } D_Z \equiv (q_i + r_i\sigma_{zi}^2(1-\rho_z^2))^{1/2}(q_j + r_j\sigma_{zj}^2(1-\rho_z^2))^{1/2}
\end{aligned} \tag{10}$$

The ranking of the *Y* versus *Z* regimes depends on team composition, the level of synergy and the relative precision of the individual measures.[18] Consider perfect

[17] All proofs are in Appendix A.

correlation of the teamwork measures: when ρ_z=1, the *Z* regime dominates the *Y* for sufficiently high synergy, and team expertise reinforces this dominance.[19] When the measures of individual effort are perfectly correlated (*i.e.*, ρ_y=1), expression (10) reduces to $\Pi^Y > \Pi^Z \Leftrightarrow 4D_Z > s^2 q_i q_j$, and either regime may dominate. In practice, one might expect a measure of teamwork to be inherently less precise than a measure of individual effort because measuring teamwork may involve more subjectivity. Even if this is so, the above analysis shows that *Z* may still dominate *Y* when synergy, team expertise and/or the correlation between the agents' teamwork measures are sufficiently high.

2.4.2.3 Regime YZ: Contracting on both individual measures

In this regime, performance measures *y* and *z* are contractible. The principal offers each agent a wage of the form $w_i = \alpha_i + \gamma_i\, y_i + \delta_i\, y_j + \kappa_i\, z_i + \lambda_i\, z_j$. Agent *i*'s problem parallels the *Y* and *Z* regimes. Taking first order conditions of the agent's certainty equivalent with respect to efforts e_i and τ_i and solving for the optimal effort levels yields $e_i^{YZ} = p_i\, \gamma_i^{YZ}$ and $\tau_i^{YZ} = q_i\, \kappa_i^{YZ}$. The principal's problem also parallels the *Y* and *Z* regimes. Solving for the optimal incentive weights and firm profit gives Lemma 2.2.

LEMMA 2.2. In regime *YZ*, the optimal effort levels, incentive weights, and firm profit are as follows:

$$\begin{aligned} e_i^{YZ} &= p_i\gamma_i^{YZ}, \tau_i^{YZ} = q_i\kappa_i^{YZ} \\ \gamma_i^{YZ} &= \gamma_i^{Y}, \delta_i^{YZ} = \delta_i^{Y}, \kappa_i^{YZ} = \kappa_i^{Z}, \lambda_i^{YZ} = \lambda_i^{Z} \\ \Pi^{YZ} &= \Pi^{Y} + \Pi^{Z} \end{aligned} \tag{11}$$

[18] The combination of synergy and performance measure precision can be interpreted as the measure's "line-of-sight", or the extent to which effort is a relatively small or large component of a performance measure. For example, when Rockwell Automation initially developed its CSMIP (Critical Success Measures Incentive Plan), it found that employees perceived some performance measures, such as worldwide inventory days, as being too far beyond their ability to control locally. Rockwell then adjusted its plan to create subsets of those measures for CSMIP, or smaller pieces of those measures that drive the longer line-of-sight measures, for the different business units (Parker *et al.* [2000: 119]).

[19] In fact, *s*>2 is sufficient because each term within the parentheses is less than one.

From equation (11), it is clear that the *YZ* regime strictly dominates each of the *Y* and *Z* regimes. As expected, the ability to induce both types of effort improves the firm's expected profit.

2.4.3 Regime X: Team output contractible

In this regime, I assume the principal can contract on team output, but individual performance measures of individual effort and teamwork are prohibitively costly to obtain.[20] The principal offers each agent a wage of the form $w_i = \alpha_i + \beta_i x$. Agent *i*'s problem is to maximize his certainty equivalent as follows:

$$\underset{e_i,\tau_i}{\text{Max}}\ \alpha_i + \beta_i\left(e_i + e_j + s(\tau_i\tau_j)^{1/2}\right) - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\sigma_x^2\beta_i^2$$

Taking first order conditions with respect to efforts e_i and τ_i and solving for the optimal effort levels yields the following:

$$e_i^X = p_i\beta_i^X, \quad \tau_i^X = \frac{s}{2}(q_i\beta_i^X)^{3/4}(q_j\beta_j^X)^{1/4} \qquad \text{for } i, j = 1,2,\ i \neq j \tag{12}$$

In the *X* regime, each team member internalizes the synergy generated together with his teammate, and adjusts his teamwork level accordingly. Agent *i*'s teamwork increases in agent *j*'s explicit incentive weight β_j and team expertise q_j. This interplay suggests the incentive weights of one's teammates can serve as a coordination device for setting appropriate effort levels for all team members. This is consistent with policies found in practice to aggressively publicize incentive plans within a firm (Parker *et al.* [2000: 193, 201]).

Given the solutions to the agents' problem, the principal maximizes:

$$\Pi^X = p_1\beta_1 + p_2\beta_2 + \frac{s}{2}(q_1\beta_1 q_2\beta_2)^{1/2} - \sum_{i=1,2}\left(c_i + \frac{r_i}{2}\operatorname{var} w_i\right) \tag{13}$$

[20] Aside from the direct expense of data collection and processing associated with individual performance measures, a firm may also face substantial indirect costs. Auriol *et al.* [2002] find that the use of individual performance measures creates an implicit incentive for a team member to sabotage his teammates to obtain a higher relative assessment of his own skill by the labor market.

Taking first order conditions with respect to β_1 and β_2 yields the following:[21]

$$p_i - \beta_i(p_i + r_i\sigma_x^{\ 2}) + \frac{s^2}{16}\left(\frac{q_i q_j \beta_j}{\beta_i}\right)^{1/2}(4 - q_j\beta_j - 3q_i\beta_i) = 0,\ \text{for } i, j = 1,2, i \neq j \quad (14)$$

Solving simultaneously for β_1 and β_2 is not tractable. However, using the implicit function theorem, one can show that β_1 is a function of β_2 and vice versa. Furthermore, $\partial\beta_i/\partial\beta_j$ takes a positive (negative) value when synergy is high (low) relative to the variance of team output. Because agent *i*'s individual effort is a multiple of his incentive weight, which is in turn a function of agent *j*'s incentive weight (*i.e.*, $e_i^X = p_i\,\beta_i^X(\beta_j^X)$), one agent's individual, *non-collaborative* effort may either be increasing or decreasing in his teammate's incentive weight. This potentially negative impact on agent *j*'s individual effort happens because both team members' teamwork responds to *both* incentive weights β_1 and β_2. This spillover effect of agent *i*'s incentive weight on agent *j* causes the principal to reduce incentive weight β_i relative to no spillover (she gets more teamwork value for her money). The principal's reduction of incentive weight β_i causes a corresponding reduction in individual effort (since $e_i^X = p_i\,\beta_i^X$).

To simplify the expressions, I impose symmetry across all agent characteristics. To identical agents, the principal offers identical incentive weights (*i.e.*, β_1=β_2=β). Solving (14) then gives the optimal incentive weights. Substituting these weights into the principal's objective function yields Lemma 2.3.

LEMMA 2.3. With identical agents, the optimal incentive weights and firm profit in regime *X* are as follows:

$$\beta_i^X = \frac{s^2q + 4p}{s^2q + 4(p + r\sigma_x^{\ 2})},\ \ \Pi^X = \frac{(s^2q + 4p)^2}{4(s^2q + 4(p + r\sigma_x^{\ 2}))} \qquad (15)$$

[21] Second order conditions verify that this is a maximum.

Firm profit in regime X increases at an increasing rate in the square of the synergy parameter (s^2). This is contrasts with the input regimes, where firm profit increases at a constant rate in s^2 (Z and YZ) or not at all (Y).

Comparison of the X and Y regimes yields:[22]

$$\Pi^X > \Pi^Y \Leftrightarrow (4p + s^2 q)\Big((4p + s^2 q)(p + r\Sigma_y{}^2) - 4p\Big) > 16 pr\sigma_x{}^2$$
$$\text{where } \Sigma_y \equiv \sigma_y{}^2(1-\rho_y{}^2)$$

As expected, the X regime generally dominates the Y regime when team output is sufficiently precise (*i.e.*, σ_x^2 low), the synergy parameter s is sufficiently high or the team expertise q is sufficiently high, because firm profit in regime Y is unaffected by these three parameters. Precision in measure y (net of correlation) favors Y over X.

Next, consider the X versus Z regimes. If the principal can implement only one performance measure and (at least some) teamwork is critical, with which of these regimes is she better off? *Ex ante*, it is not clear whether one regime will consistently outperform the other. Both regimes successfully induce teamwork; the X regime relies on an output measure, whereas the Z regime uses an input measure. Team members directly internalize the synergy in the X regime via team output (measure x). In contrast, the principal induces the interdependency between agents via incentive weights in the Z regime.

Comparing the firm profit under the X and Z regimes yields:

$$\Pi^X > \Pi^Z \Leftrightarrow s^2 r\Sigma_z{}^2 + 4\frac{p}{q}(q + 2r\Sigma_z{}^2) + 16\frac{p^2}{s^2 q^2}(q + r\Sigma_z{}^2) > 4r\sigma_x{}^2 \tag{16}$$
$$\text{where } \Sigma_z{}^2 \equiv \sigma_z{}^2(1-\rho_z{}^2)$$

Clearly, whenever one measure is more precise than the other, the associated regime is more likely to dominate. When synergy is sufficiently low (*e.g.*, $s \to 0$), the X regime dominates because it also contains an individual effort component. As synergy

[22] To facilitate comparison across regimes, the Y and Z regimes are stated here with identical agents.

(*i.e.*, the sensitivity of team output, x, to teamwork) increases, the X regime eventually dominates because profit increases at an *increasing* rate in s^2, versus increasing at a constant rate in s^2 in the Z regime. As a result, when synergy is sufficiently high (*e.g.*, $s \to \infty$), the X regime dominates. However, for intermediate values of synergy, firm profit in regime Z can increase faster in synergy than does firm profit in regime X, which in turn increases faster in synergy than does regime Y (*i.e.*, $\partial\Pi^Z/\partial s > \partial\Pi^X/\partial s > \partial\Pi^Y/\partial s = 0$).

Not surprisingly, task expertise favors the dominance of regime X over Z because only regime X benefits from more efficiency in individual effort. It is less obvious which regime would benefit more from an increase in team expertise q. Note that as q increases, the left-hand side of (16) decreases. More generally, taking the derivative of the difference between firm profit in regimes X and Z with respect to q yields:

$$\frac{\partial}{\partial q}(\Pi^X - \Pi^Z) = \frac{s^2}{4}\frac{(s^2 q + 4p)^2 + 8r\sigma_x{}^2(s^2 q + 4p)}{(s^2 q + 4(p + r\sigma_x{}^2))^2} - \frac{s^2}{4}\frac{q(q + 2r\Sigma_z{}^2)}{(q + r\Sigma_z{}^2)^2}$$

$$\Rightarrow \frac{\partial}{\partial q}(\Pi^X - \Pi^Z) > 0 \Leftrightarrow s^2 + \frac{p}{q} > 4\frac{\sigma_x{}^2}{\Sigma_z{}^2}$$

This parsimonious expression shows the interplay between synergy, precision and team composition in determining which regime sees firm profit increase faster in team expertise. *Ceteris paribus*, when team composition determines the direction of the inequality, an increase in team expertise benefits a task-focused team (*i.e.*, relatively more task expertise) more in the X regime, whereas a team-focused team benefits more in the Z regime. These points are summarized in Proposition 2.1.

PROPOSITION 2.1. With identical agents, the relative ranking of firm profit in the team output-based X regime versus the teamwork input-based Z regime is as follows:

1. If synergy is sufficiently high *or* low, firm profit is higher in the X regime.
2. For intermediate synergy, either regime can dominate.

3. Increases in task expertise p favor the X regime (*i.e.*, increases in p benefit regime X more than regime Z).
4. Increases in team expertise q favor regime X (regime Z) when the expression $s^2 + \frac{p}{q} - 4\frac{\sigma_x{}^2}{\Sigma_z{}^2}$ is positive (negative).

Proposition 2.1 demonstrates that determining whether a noisy measure of team output or teamwork input is more valuable to the firm depends not only on the precision of the measures, but also on team composition (*i.e.*, team versus task expertise) and the degree of synergy. Thus, in some situations, either of the contradictory recommendations by Lawford [2003] (*don't* reward on individual measures) and Parker *et al.* [2000] (*do* reward on individual measures) can be in the firm's best interest.

Finally, compare the X and YZ regimes:

$$\Pi^X > \Pi^{YZ} \iff \frac{(s^2q + 4p)^2}{4(s^2q + 4(p + r\sigma_x{}^2))} > \frac{p}{p + r\Sigma_y{}^2} + \frac{s^2}{4}\frac{q^2}{(q + r\Sigma_z{}^2)} \tag{17}$$

Note that whenever the Z regime dominates the X regime, the YZ regime also dominates X. Some refinements from adding measure y include: (1) if $\frac{s^2}{4} + \frac{p}{q} > \frac{\sigma_x{}^2}{\Sigma_z{}^2}$, then team expertise q favors regime X over YZ, and (2) if $\frac{s^2q + 4p}{2} > \frac{\sigma_x{}^2}{\Sigma_z{}^2}$, then the synergy s favors regime X over YZ. [23]

2.4.4 Regime XY: Mix of team and individual performance measures contractible

Given the ambiguous dominance of stand alone team versus individual performance measures, I next look at a mix of these two measures. In this regime, I assume the principal has both team output and one type of individual performance measure available for contracting purposes. For tractability, assume that the measure of

[23] These are both sufficient but not necessary conditions. See Appendix A for the necessary and sufficient conditions.

individual effort is available for contracting but that a measure of teamwork is prohibitively costly to obtain. The principal offers each agent a wage of the form $w_i = \alpha_i + \beta_i x + \gamma_i y_i + \delta_i y_j$. Agent i's problem is to maximize his certainty equivalent as follows:

$$\max_{e_i, \tau_i} \ \alpha_i + \beta_i (e_i + e_j + s(\tau_i \tau_j)^{1/2}) + \gamma_i e_i + \delta_i e_j - \frac{1}{2p_i} e_i^2 - \frac{1}{2q_i} \tau_i^2 - \frac{r_i}{2} \sigma_x^2 \beta_i^2$$

Taking first order conditions with respect to efforts e_i and τ_i and solving for the optimal effort levels yields the following:

$$e_i^{XY} = p_i (\beta_i^{XY} + \gamma_i^{XY}), \quad \tau_i^{XY} = \frac{s}{2}\left(q_i \beta_i^{XY}\right)^{3/4} \left(q_j \beta_j^{XY}\right)^{1/4} \qquad \text{for } i, j = 1,2, \ i \neq j$$

Given the solutions to the agents' problem, the principal maximizes:

$$\Pi^{XY} = \frac{s}{2}(q_1 q_2 \beta_1 \beta_2)^{1/2} + \sum_{i=1,2} \left(p_i (\beta_i + \gamma_i) - c_i - \frac{r_i}{2} \operatorname{var} w_i \right) \tag{18}$$

First order conditions with respect to β_i, γ_i and δ_i are therefore:

$$\begin{aligned} & p_i - \beta_i^{XY} (p_i + r_i \sigma_x^2) + \frac{s^2}{16} \left(\frac{q_i q_j \beta_j^{XY}}{\beta_i^{XY}} \right)^{1/2} (4 - q_j \beta_j^{XY} - 3 q_i \beta_i^{XY}) - \gamma_i^{XY} = 0 \\ & p_i - \beta_i^{XY} - \gamma_i^{XY} (p_i + r_i \sigma_{yi}^2) - \delta_i^{XY} \rho_y r_i \sigma_{yi} \sigma_{yj} = 0 \\ & -\delta_i^{XY} r_i \sigma_{yj}^2 - \gamma_i^{XY} \rho_y r_i \sigma_{yi} \sigma_{yj} = 0 \end{aligned} \tag{19}$$

Solving for γ_i and δ_i is straightforward. However, as in the *X* regime, solving simultaneously for β_i and β_j is not tractable. To obtain solutions in closed form, I again impose symmetry across agents. To identical agents, the principal offers identical incentive weights (*i.e.*, $\beta_i = \beta_j = \beta$). Solving (19) with these values yields the optimal incentive weights and firm profit, as shown in Lemma 2.4.

LEMMA 2.4. With identical agents, the optimal incentive weights and firm profit in regime *XY* are as follows:

$$\gamma_i^{XY} = \frac{p_i(1-\beta_i^{XY})}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)}, \quad \delta_i^{xy} = -\gamma_i^{XY}\frac{\rho_y\sigma_{yi}}{\sigma_{yj}}, \quad \text{for } i, j = 1,2,\ i \neq j$$

$$\beta_i^{XY} = \frac{s^2pq + r\Sigma_y^2(4p+s^2q)}{p(s^2q+4r\sigma_x^2) + r\Sigma_y^2(4p+s^2q+4r\sigma_x^2)}, \text{ where } \Sigma_y^2 \equiv \sigma_y^2(1-\rho_y^2) \quad (20)$$

$$\Pi^{XY} = \frac{1}{4}\left(\frac{16p^2r\sigma_x^2 + (4p+s^2q)(pqs^2 + r\Sigma_y^2(4p+s^2q))}{p(s^2q+4r\sigma_x^2) + r\Sigma_y^2(4p+s^2q+4r\sigma_x^2)}\right)$$

As expected, *XY* dominates the *X* and *Y* regimes. Next, consider the difference between the *XY* and *YZ* regimes.

$$\Pi^{XY} > \Pi^{YZ}$$

$$\Leftrightarrow (4p+s^2q)\left(4pD_YD_Z + s^2qD_Yr\Sigma_z^2\right) > 2pD_Z(4r\sigma_x^2 + 4p + s^2q) + 4s^2q^2D_Yr\sigma_x^2 \quad (21)$$

where $D_Y \equiv p + r\Sigma_y^2$, $D_Z \equiv q + r\Sigma_z^2$

Figure 2.1 illustrates that whether a firm is better off with a blend of performance measures (regime *XY*) or with individual measures alone (regime *YZ*) depends in part on the relative net precisions of team output and the measure of teamwork.

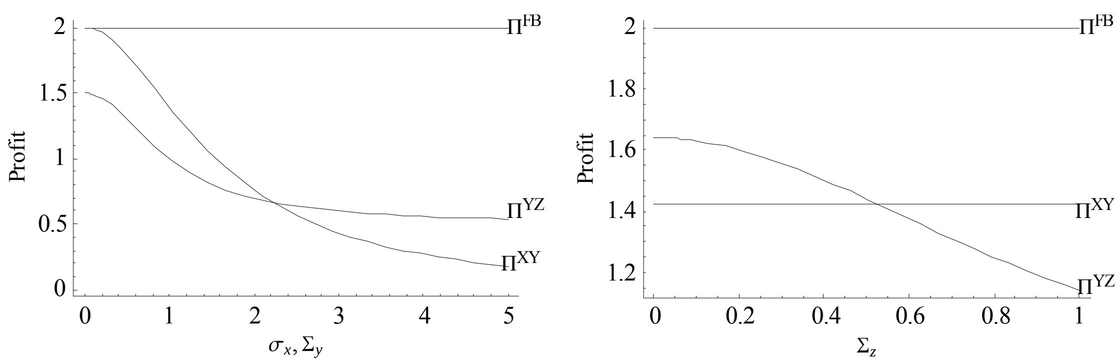


Figure 2.1. Firm profit across regimes as a function of σ_x and Σ_y (p=q=r=1, s=2, Σ_y^2=σ_x^2, Σ_z^2=1) and as a function of Σ_z (p=q=r=1, s=2, Σ_y^2=3/4, σ_x^2 =1).

However, Figure 2.1 does not tell the whole story. Figure 2.2 shows that team composition also helps determine which regime dominates.

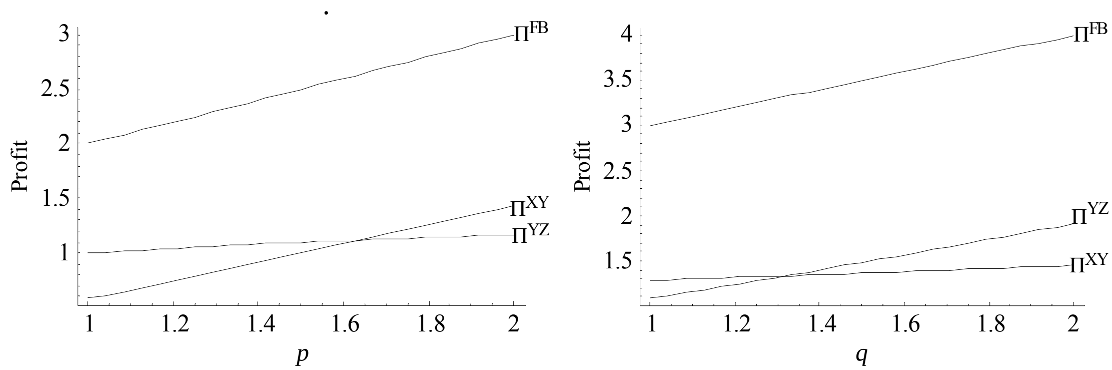


Figure 2.2. Firm profit as a function of task expertise p (q=1,r=1, s=2, σ_x^2 =5, Σ_y^2=Σ_z^2=1) and as a function of team expertise q (p=2,r=1, s=2, σ_x^2 =5, Σ_y^2=Σ_z^2=1).

In both regimes *XY* and *YZ*, firm profit increases in task/team expertise, the net precision of measure *y* (*i.e.*, Σ_y^{-2}) and in synergy parameter *s*. *Ceteris paribus*, either regime may dominate when synergy is low (*e.g.*, $s \to 0$): expression (21) reduces to $\Pi^{XY} > \Pi^{YZ} \Leftrightarrow p(2D_Y - 1) > r\sigma_x^{\ 2}$. When synergy is sufficiently high (*i.e.*, the s^4 term on the left-hand side of (21) dominates), the *XY* regime is favored. Intermediate values of synergy and the precision of the measure *y* are interrelated and ambiguous in the ranking of the *XY* and *YZ* regimes.

PROPOSITION 2.2. With identical agents, the relative ranking of firm profit in the blend of team and individual measures regime, *XY*, versus the individual measures only regime, *YZ*, is as follows:

1. If synergy is sufficiently high, firm profit is higher in the *XY* regime.
2. If synergy is sufficiently low, either regime may have higher profit. Which regime dominates depends on task expertise and the net precision of measures *x* and *y*.
3. There exist conditions under which the firm profit is higher under regime *YZ* than *XY*.

These results are generally consistent with some of the main takeaways of Parker *et al.* [2000], an empirical study of team incentive contracts found in practice. First, customize the incentive plan to the organization in question. Second, align the incentive plan with business objectives by choosing the right performance measures: the key to success is through teamwork.

2.5. CONCLUSION

This chapter introduces interdependencies between team members, *i.e.*, team synergy, and addresses two research questions. First, for an exogenously given set of performance measures, how can a firm best induce teamwork and reap the benefits of synergy using linear incentive contracts? Second, if a firm can afford only one (or only one type of) performance measure, with which measure is the firm better off?

This analysis seeks to reconcile contradictory guidance to firms who seek to design an incentive plan to induce teamwork and achieve synergy. Lawford [2003] recommends that firms *not* single out individual team members for special acknowledgement, to prevent competitiveness that can damage a collaborative environment. On the other hand, Parker *et al.* [2000] advise that firms *do* reward individual performance. I find that, given the availability of a single performance measure, either of these approaches may dominate the other. This analysis therefore may have practical implications for a firm's choice of performance measures when potential team synergy is present.

This chapter examines one specific form of team synergy. I assume that team members choose two different types of effort, individual effort and teamwork. Synergy is an increasing and concave multiplicative function of teamwork. Individual performance measures do not capture this synergy. I vary the availability of team and individual performance measures and ascertain the impact on optimal linear incentive weights that induce individual effort and teamwork to achieve synergy. I find that when team members influence each other's marginal productivity via teamwork, each agent's teamwork choice depends not only on his own incentive weights and team expertise, but also on the incentive weights and team expertise of his teammate. Further, I find that firm profit can be higher in either a team-output-based or a teamwork-input-based contracting regime. An increase in team expertise may benefit a firm with a *task*-focused team (*i.e.*, relatively more task expertise than team expertise) more in a team-*output*-based contracting regime, whereas a firm with a *team*-focused team may benefit more in a teamwork-*input*-based contracting regime. Finally, firm profit can be higher when contracting only on two types of input-based individual performance measures than when contracting on a blend of team output and one type of input measure. The most valuable performance measure(s) to the firm for contracting purposes depends not only on the precision of the measures, but also on team composition and the degree of synergy.

Additional modeling choices inherent in the LEN structure limit the generalizability of the results. Most notably, the exogenous assumption of linear wage contracts, while consistent with prior literature, precludes the consideration of optimal contracts. In addition, I assume constant absolute risk aversion (negative exponential utility function) of both agents and normally distributed error terms. For simplicity, I assume no correlation between error terms of different types of performance measures. These assumptions provide the necessary tractability for modeling complex constructs such as teamwork and synergy.

Chapter 3: Differential Career Concerns and Team Synergy

3.1. Introduction

Career concerns arise when an agent expends incremental effort in an attempt to influence the labor market's assessment of his ability, thus increasing his future wages. Analytic models predict, and empirical evidence supports, that junior workers have greater career concerns than those near the end of their careers.[24] That is, differential career concerns induce different levels of effort across worker types.

Team synergy occurs when the total output of a team, working collaboratively, exceeds the sum of the output of the team members working individually. Chapter 2 of this dissertation finds that in a synergistic environment, the level of teamwork each agent chooses depends not only on his own characteristics, but also on his teammates' characteristics. These results suggest that differential career concerns among team members may also influence the decision of how much teamwork to perform.

This chapter examines how these two forces affecting effort – differential career concerns and team synergy – interact in environments where different performance measures are observed. Specifically, I explore the impact on a team member's non-collaborative and collaborative effort choices and on the resulting firm profit of (i) the relative importance of synergy, (ii) a team member's own career concerns, (ii) his teammate's career concerns, (iii) the interaction between the career concerns of different team members, and (iv) the interaction between synergy and career concerns. For example, what impact does the relative lack of career concerns of a senior team member

[24] Gibbons and Murphy [1992] analytically develop and empirically test a prediction that junior workers have greater career concerns, and hence receive less explicit incentives, than those near the end of their careers. Kaarboe and Olsen [2004] model a multi-task environment where junior workers have greater career concerns (which potentially conflict with the principal's interests), and hence they may receive *more* explicit incentives, than senior workers.

have on his own effort choices, on his teammate's effort choices, and ultimately on profit? When is a firm better off with a team of all junior, all senior, or a mix of agent types? The results of this chapter may help explain why teams of junior or senior people are in more demand in some environments than in others.

These questions are important to firms and researchers alike because teams – comprised of individuals with varying career concerns – have become omnipresent in the workplace, with a correspondingly large economic impact on firms. Presumably the purpose of these teams is to achieve synergy: if not, no team is needed. This research is especially timely due to the upcoming shift in seniority in the work force as an expected wave of retirements begins in the next few years.[25] In its 2005 survey of human resources executives, Deloitte Consulting found that more than 60 percent of the 123 respondents said baby boomer retirement poses the greatest threat to business performance over the next three years. This study examines one aspect of the upcoming change in workforce composition and its associated implications for team synergy, which is expected to have a major economic impact on firms.

To address the research questions, I employ a two-period model of a firm that hires two agents to work together on a team.[26] I vary the type of performance measure available to the firm: (1) an output measure that includes a synergy component, (2) a pair of individual input measures with no such synergy component, or (3) both input and output measures. Further, I assume that the agents who comprise the team have varying degrees of career concerns. To focus the analysis on career concerns, I assume that the

---

[25] The first wave of baby boomers turns 62 in three years, the average retirement age in North America, Europe and Asia. According to the Deloitte survey "It's 2008: Do You Know Where Your Talent Is? Why Acquisition and Retention Strategies Don't Work," one-third of U.S. companies expect to lose 11 percent or more of their current workforce to retirements by 2008.

[26] Given the difficulty of directly observing either career concerns *or* synergy in practice, an analytic model is an appropriate tool to identify and analyze these effects, generating testable empirical predictions.

second period measures have no effort component; this assumption eliminates ratchet effects from the analysis and allows more parsimonious solutions.

I model a short-term contracting environment where the performance measures are observable but exogenously non-contractible.[27] This assumption not only increases tractability, but it also isolates career concerns as the sole motivator of individual effort and teamwork, permitting an exclusive focus on analyzing the interaction between differential career concerns and team synergy.

For simplicity, the principal hires the team to perform a one-period task, and accordingly all contracts are for one period only (i.e., mandatory rotation). This assumption removes the firing decision and the potential for an adverse selection problem in the labor market while preserving career concerns. Examples of one-period team employment with career concerns include startup/ transitional/ turnaround specialist executive teams who are hired only for the duration of a certain firm life-cycle phase, or one-time consulting and other professional team engagements; the outcomes of these engagements have a direct impact on future compensation.

The results indicate that, ceteris paribus, firms prefer more synergy to less. A firm whose team is comprised of agents of equal seniority is strictly better off with junior team members (i.e., workers with higher career concerns), and synergy strengthens the positive effect of career concerns. Intuitively, this happens because, in the absence of explicit incentives, career concerns are required to induce any effort at all, and synergy is required to produce positive collaborative effort. However, for heterogeneous teams,

[27] In chapter 4, I model the same regimes with contractible performance measures. However, the assumption of non-contractible performance measures is not uncommon in the career-related literature (e.g.,Holmstrom and Ricart i Costa [1986], Jeon [1996], Dewatripont, Jewittt, & Tirole [1999], Berck and Lipow [2000] and Hoffler and Sliwka [2003]). When all performance measures are contractible, prior research suggests that firms modify the explicit incentives to undo part or all of the impact of implicit incentives (e.g., Gibbons and Murphy [1992], Autrey, Dikolli, & Newman [2005], Kaarboe and Olsen [2004]).

stronger career concerns are not necessarily better: the interaction between agents' career concerns can have either a beneficial or adverse impact on firm profit. The results indicate that the impact on firm profit depends on the level of synergy, the precision of performance measures and the degree of team heterogeneity.

This chapter builds on a small analytic literature of teams and career concerns.[28] However, there are few multiple-agent and/or multiple-task career concerns models. Two such single-action, multiple-agent models are Meyer and Vickers [1997] and Jeon [1996]. Meyer and Vickers [1997] analyze a quasi-two-agent model in which firms separately maximize across each agent's individual and contractible output (net of wages), rather than maximizing collective team output. There is no team aspect; the purpose of multiple agents is to provide relative performance information only. The authors explore the interaction between relative performance evaluation and implicit incentives. Jeon [1996] models a two-agent team in which the only observable measure is a non-contractible aggregate team output measure. Output is additively separable in the agents' effort level, ability and a shock term; there is no synergy component. He finds that a team comprised of a mix of older and younger workers is more efficient than a team from a single generation.

Turning to the multiple-task, single-agent research stream, two such papers are Dewatripont *et al.* [1999] and Kaarboe and Olsen [2004]. Dewatripont *et al.* [1999] study the impact of information structures on career concerns in a generalized non-contractible setting. They find that improved information may increase *or* decrease incentives. Kaarboe and Olsen [2004] model a single agent whose complementary tasks

---

[28] The larger career concerns literature is consists primarily of single-agent, single-action models, including: Carrillo [2003], Hoffler and Sliwka [2003], Tadelis [2002], Andersson [2002], Zabojnik [2001], Berck and Lipow [2000], Nagar [1999], Jeon [1998], Farber and Gibbons [1996], Nagarajan, Sivaramakrishnan and Sridhar [1995], Senbongi and Harrington [1995], Sridhar [1994], Bernhardt and Scoones [1993],and Gibbons and Murphy [1992]. Earlier career concerns work is surveyed by Borland [1992].

produce a synergy-like component in output, and a principal who has more information about the agent's type than the labor market. In contrast to prior career concerns literature, in which firms reduce explicit incentives for career concerns, they find explicit and implicit incentives can be complements. Thus, a junior worker (i.e., higher career concerns) might receive *more* explicit incentives than a senior worker.

One paper, Auriol *et al.* [2002], models a multiple-agent, multiple action setting. In their model, an individual, contractible performance measure is observed for each of two team members. Each individual measure is an additive function of an agent's own individual effort and his *teammate's* teamwork, but, as team output is simply the sum of the individual performance measures, there is no synergy in the model. Auriol *et al.* [2002] find that the agent's implicit incentives are strictly negative – each agent has a "sabotage" incentive because for a given realization of team output in period 1, a higher realization of one's teammate's period 2 individual performance means a correspondingly lower expectation of the agent's own ability.

None of these models, however, includes an interactive team synergy component. One such model is presented in chapter 2: a single-period model in which two agents each choose two types of effort, collaborative effort and non-collaborative effort. The team output of agents who choose to collaborate with each other exceeds the output when agents do not work together. This extra output, or "synergy," is produced by an interactive term: each agent's teamwork increases their teammate's marginal productivity.

This chapter bridges all three research streams. I combine the interactive synergy term from chapter 2 with the team career concerns model in Auriol *et al.* [2002] and the team generational composition from Jeon [1996], modeled as differential career concerns.

The rest of the chapter is organized as follows. Section 3.2 introduces the model. Section 3.3 discusses the results in each performance measurement regime. Section 3.4 discusses limitations and concludes.

3.2. THE MODEL

Consider a two-period model where each period a risk-neutral principal hires two risk- and effort-averse agents for a one-period task. Each agent has ability a_i, which is constant across periods and firms.[29] In the first period, agent *i* (i=1,2) chooses individual non-negative effort, e_i, and teamwork, τ_i. In period 1, team output x_1 is a linearly additive function of agent abilities a_1 and a_2, individual efforts e_1 and e_2, a team synergy term, $s(\tau_1\tau_2)^{1/2}$, and a transient shock, ε_{x1}. In period 2, team output consists only of ability and a transient shock, as follows:[30]

$$x_1 = a_1 + a_2 + e_1 + e_2 + s(\tau_1\tau_2)^{1/2} + \varepsilon_{x1}$$
$$x_2 = a_1 + a_2 + \varepsilon_{x2}$$

where s>0 is an exogenous parameter known to all parties that represents the strength of the synergy. The individual performance measures, if they exist, are linearly additive functions of ability a_i, individual effort e_i, and teamwork τ_i and a transient shock, ε_{yit}, as follows:

$$\left.\begin{aligned} y_{i1} &= a_i + e_{it} + f_{ii}\tau_i + f_{ij}\tau_j + \varepsilon_{yi1} \\ y_{i2} &= a_i + \varepsilon_{yi2} \end{aligned}\right\} \quad i = 1,2$$

This functional form reflects that disentangling the agent's individual effort from his teamwork may not be possible. For example, billable hours may reflect a combination of time spent in individual effort *and* helping one's teammates. When this is the case, only an aggregate measure of the two types of effort is observable, and the parameter $f_{ii}\in[0,1]$ represents the weight of agent *i*'s teamwork (relative to his individual

---

[29] To avoid confusion, the index *i* always refers to which agent, and the index *t* indicates which period.

[30] Period 2 team output could be $x_{i2} = a_i + \varepsilon_{xi2}$; that is, the team need not exist in period 2.

effort) on his own measure. Further, a team member's individual performance measure may increase when his teammate assists him. The parameter $f_{ij} \in [0,1]$ represents the weight of agent *j*'s teamwork (relative to agent *i*'s individual effort) on agent *i*'s individual performance measure.[31]

For simplicity, let the agents' abilities be independent of each other and of all transient shocks; likewise, let all shock terms be independent. The random variables are normally distributed as follows:

$$a_i \sim N(\mu_i, k_i\sigma^2), \quad \varepsilon_{xt} \sim N(0, m\sigma^2), \quad \varepsilon_{yit} \sim N(0, n\sigma^2)$$
$$x_1 \sim N(\mu_1 + \mu_2 + e_1 + e_2 + s(\tau_1\tau_2)^{1/2},\ (k_1 + k_2 + m)\sigma^2)$$
$$x_2 \sim N(\mu_1 + \mu_2, (k_1 + k_2 + m)\sigma^2)$$
$$y_{i1} \sim N(\mu_i + e_i + f_{ii}\tau_i + f_{ij}\tau_j, (k_i + n)\sigma^2)$$
$$y_{i2} \sim N(\mu_i, (k_i + n)\sigma^2)$$

The variable μ_i represents the *ex ante* expected value of agent *i*'s ability. The variable k_i represents the amount of variance in team output that relates to ability and can be interpreted as agent *i*'s degree of career concerns: higher values of k_i represent more diffuse priors about the agent's ability. The variables *m* and *n*, respectively, represent the performance measure variance related to the transient shock in the team output and individual performance measures.

Each agent's cost of effort is a twice-differentiable convex increasing function of individual effort and teamwork. For simplicity, assume the costs of each type of effort are additively separable. In period 2, there is no effort and accordingly no effort cost. In period 1, total cost of effort c_i has the functional form:

$$c_i = \frac{1}{2}e_i^2 + \frac{1}{2}\tau_i^2, \quad \text{for } i = 1, 2$$

[31] I restrict the upper bound on f_{ii} and f_{ij} to ensure positive expected firm profit in all regimes (specifically, in the regime where only individual measures are observable). It seems intuitive that one or both of the f_{ii} and f_{ij} parameters will be zero in certain cases. For generality, I include both parameters whenever tractable.

Each agent has a constant absolute risk aversion (CARA) utility function:

$$\left.\begin{array}{l} u_i^N \equiv -\exp\{-r_i[w_{i1}^N + w_{i2}^N - c_i^N]\} \\ u_{i2}^N \equiv -\exp\{-r_i[w_{i2}^N - c_i]\,|\, p_1^N\} \end{array}\right\} i = 1,2 \tag{22}$$

where $r_i \in (0,\infty)$ is the Arrow-Pratt measure of absolute risk aversion for agent *i* and p_1^N consists of the observable period 1 performance measures in regime *N*.[32]

Because there is no incentive pay (due to non-contractible measures), the principal can only offer the agents a fixed wage w_{it} (i,t=1,2), which must meet or exceed the agent's outside opportunities to induce the agent to accept the principal's offer.

All parties – the principal and all labor market firms – have access to all observable period 1 performance measures. Conditional on observing these realizations, the principal and labor market update their priors about each agent's ability. The labor market is perfectly competitive and sets each agent's reservation utility as follows:

$$U_{i1}^N \equiv E[a_i] = \mu_i \text{, and } U_{i2}^N(p_1^N) \equiv E[a_i \,|\, p_1^N], \;\; i = 1,2 \tag{23}$$

3.3. ANALYSIS

All regimes are solved by backward induction: the period 2 problem is solved first, and then the period 1 problem is solved given the period 2 solutions. The agent makes no choices in period 2. The period 2 principal is risk neutral and maximizes expected profit:

$$\underset{\alpha_{i2},\, i=1,2}{\text{Max}} \; \Pi_2 \equiv E[x_2 \,|\, p_1^N] - E[w_{12} \,|\, p_1^N] - E[w_{22} \,|\, p_1^N]$$

$$\text{subject to} \quad \text{(PC)} \quad E[w_{i2} \,|\, p_1^N] \geq U_{i2}(p_1^N), \; i = 1,2$$

To obtain the services of the agents, the principal must offer each agent at least the utility he could obtain in the labor market in period 2 (conditional on period 1 observed realizations), represented by the participation constraint (PC).

---

[32] Regime $N \in \{X, Y, XY\}$ refers to the regime where only performance measures *x* or *y* or both, respectively, are observable to the principal and the labor market.

In period 2, the principal precisely meets each agent's participation constraint, so $E[w_{i2} \mid p_1^N] = U_{i2}(p_1^N)$. Thus, the offered wage equals the reservation wage, which by (23) equals the agent's revised expected ability:

$$\alpha_{i2}^N = U_{i2}^N(p_1^N) \equiv E[a_i \mid p_1^N], \quad i = 1,2 \tag{24}$$

In period 1, each agent chooses individual effort and teamwork to maximize his expected utility, taking into account the effect of this effort on the period 2 wage. Each agent's period 1 wage is simply the *ex ante* expectation about his ability, μ_i:

$$\alpha_{i1}^N = U_{i1}^N \equiv E[a_i] = \mu_i, \quad i = 1,2 \tag{25}$$

Substituting $\alpha_{i2}{}^N$ from (24) into agent i's expected utility from (22) and rewriting utility in the certainty equivalent form yields:[33]

$$ACE_i^N \equiv \mu_i + E[a_i \mid p_1^N] - c_i^N, \quad i = 1,2 \tag{26}$$

The period 1 principal is also risk-neutral and maximizes expected profit:

$$\left.\begin{aligned} &\underset{\alpha_{i1}, e_i, \tau_i}{\text{Max}} \ \Pi_1 \equiv Ex_1 - Ew_{11} - Ew_{21} \\ &\quad \text{subject to (PC) } Ew_{i1} + Ew_{i2} - c_i - \frac{r}{2}\text{var}(w_{i1} + w_{i2}) \geq U_{i1}^N + EU_{i2}^N \\ &\qquad\qquad \text{(IC) } e_i, \tau_i \in \arg\max Eu_i^N \end{aligned}\right\} \quad i = 1,2$$

In period 1, each agent's action must maximize his utility, represented by the incentive compatibility constraint (IC). Note that each agent's participation constraint maximizes cumulative period 1 and period 2 utility, whereas the period 1 principal only compensates the agent for period 1. However, the period 2 principal's solution is anticipated by all parties, and so each agent's (binding) participation constraint can be rewritten as follows:

$$Ew_{i1} = 2\mu_i + c_i + (r/2)\,\text{var}(w_{i1} + w_{i2}) - \alpha_{i2}, \quad i = 1,2 \tag{27}$$

[33] The agent's utility can be expressed in this mean-variance form due to the LEN assumptions: a Linear wage, Negative Exponential agent utility and Normally distributed performance measures.

Ex ante, the labor market's belief about the agent's ability is the same for period 1 and period 2, (i.e. U_{i1}^N = EU_{i2}^N = μ_i). Thus, when no measures are contractible (i.e., w_{i1}=α_{i1}), the updating of beliefs incorporated in the period 2 fixed wage provides the *only* source of compensation to the agent influenced by period 1 effort. The period 1 principal's problem can be rewritten as:

$$\Pi_1 = Ex_1 - \sum_{i=1,2} \left[2\mu_i + c_i + (r/2)\,\text{var}(w_{i1} + w_{i2}) - \alpha_{i2} \right] \tag{28}$$

3.3.1 Team output measure observable (Regime *X*)

In this regime, period 1 team output is observable but not contractible, and no individual performance measures exist. All parties update their beliefs about the agent's type based on the observed period 1 team output, x_1, and the period 2 fixed wage, α_{i2}, is adjusted accordingly:

$$\alpha_{i2}^X = E[a_i \mid x_1] = \mu_i + \frac{k_i}{k_1 + k_2 + m}(x_1 - \hat{x}_1) \tag{29}$$

where $\hat{x}_1$ is the expected value of x_1 given the principal's and labor market's conjectures about the agents' effort choices, $\hat{e}_i$ and $\hat{\tau}_i$ (i.e., $\hat{x}_1 = \mu_1 + \mu_2 + \hat{e}_1 + \hat{e}_2 + s(\hat{\tau}_1\hat{\tau}_2)^{1/2}$). To solve for each agent's period 1 effort choices, substitute (29) into (26), take first order conditions and solve, which yields Lemma 3.1.

LEMMA 3.1. When non-contractible team output x is the only observable performance measure, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (i,j=1,2, i≠j):

$$e_i^X = \frac{k_i}{k_1 + k_2 + m}, \quad \tau_i^X = \frac{s}{2}\frac{k_i^{3/4}k_j^{1/4}}{(k_1 + k_2 + m)} \tag{30}$$

PROOF: All proofs are in Appendix B.[34]

[34] There is no subgame problem in this or any regime modeled. That is, neither agent has a profitable deviation when his teammate chooses the equilibrium effort level. Furthermore, although some of the regimes have multiple equilibria, there is only one equilibrium in each regime with positive effort levels: the other equilibria have zero, negative or complex effort levels.

Even in the absence of explicit incentives, each agent's career concerns induce positive levels of both individual effort and teamwork (for s>0). As expected, individual effort is not a function of the synergy parameter. Further, it is straightforward from (30) that agent i's individual effort increases in his own career concerns k_i and decreases in his teammate's career concerns k_j.[35] However, an increase in agent j's career concerns may either increase or decrease the direct career effect (i.e., the marginal impact of a change in agent i's career concerns on agent i's individual effort) for i,j=1,2, $i \neq j$:

$$\frac{\partial^2 e_i^X}{\partial k_i \partial k_j} = \frac{k_i - k_j - m}{\left(k_1 + k_2 + m\right)^3}$$

Thus, a more senior agent ($k_i < k_j$) has his career effect dampened by increases in his junior partner's career concerns, and a sufficiently junior agent ($k_i > k_j+m$) has his career effect enhanced by increases in his senior partner's career concerns. When agents have equal seniority ($k_i = k_j$), individual effort increases in k at a decreasing rate.[36]

Each agent's teamwork level incorporates not only his own career concerns (a "direct" career effect), but also those of his teammate (an "indirect" career effect), due to the interdependent synergy term in team output. In this regime, teamwork increases at a constant rate in the synergy parameter.

The interdependency between agents produces a more complicated relationship between teamwork and each agent's degree of career concerns. Agent i's teamwork is affected by changes in career concerns as follows (i,j=1,2, $i \neq j$):

$$\frac{\partial \tau_i^X}{\partial k_i} = \frac{s}{8} \frac{\left(3(k_j + m) - k_i\right) k_j^{1/4}}{k_i^{1/4} (k_1 + k_2 + m)^2}, \quad \frac{\partial \tau_i^X}{\partial k_j} = \frac{s}{8} \frac{\left(k_i - 3k_j + m\right) k_i^{3/4}}{k_j^{3/4} (k_1 + k_2 + m)^2}$$

35 This decrease is in essence free-riding on his teammate's career concerns, and it occurs because the individual efforts of each agent are substitutes. Note that the teamwork choices are complements, so free-riding per se does not occur.

36 One example of a team with agents at the same career level (a "peer-to-peer" team) is a cross-functional team, such as an executive team.

First, consider the direct career effect, $\frac{\partial}{\partial k_i}$, on agent *i*'s teamwork τ_i^X. Agent *i*'s teamwork choice increases in his own career concerns if and only if he is sufficiently senior ($k_i<3(k_j+m)$). Equivalently, with a sufficiently imprecise team output measure ($m>(k_i/3)–k_j$), all agents choose less teamwork as their own career concerns increase, because the imprecision reduces the impact of any incremental teamwork.

A senior agent's ($k_i<k_j$) teamwork increases in his own career concerns (i.e., increases as he becomes less senior), whereas a junior agent's ($k_i=3(k_j+m)$) teamwork decreases in his career concerns. This counterintuitive *negative* direct career effect happens because of countervailing career effects and the interdependency between agents. Note that one agent's teamwork is a function of both team members' individual effort (specifically, $\tau_i=(s/2)e_i^{3/4}e_j^{1/4}$). Recall that each agent's individual effort increases in his own (direct) career concerns and decreases in the (indirect) career concerns of his teammate. The negative indirect career effect dominates for a junior person whereas the positive direct career effect dominates for a senior person.[37]

Second, consider the indirect career effect on teamwork, $\frac{\partial \tau_i^X}{\partial k_j}$. Agent *i*'s teamwork choice increases in his teammate's career concerns if and only if his teammate is sufficiently senior ($k_j<(k_i+m)/3$). Hence, a sufficiently junior team member ($k_i>3k_j-m$) responds positively to his more senior teammate's career concerns. Or, alternatively stated, with a sufficiently imprecise team output measure ($m>k_i+3k_j$), all agents choose more teamwork as their teammate's career concerns increase. The intuition parallels the direct career effect above.

---

[37] To see this, consider the product rule: $D(ab)=a'b+ab' \Rightarrow \frac{3e_j^{1/4}}{4e_i^{1/4}}\left(\frac{k_j+m}{A}\right)-\frac{e_i^{3/4}}{4e_j^{3/4}}\left(\frac{k_i}{A}\right)$, where $A=(k_1+k_2+m)^2$.

Next, consider the cross-partial effects for teamwork. Note that both $\frac{\partial \tau_i^X}{\partial k_i}$ and $\frac{\partial \tau_i^X}{\partial k_j}$ increase or decrease at a constant rate in synergy, so synergy reinforces the marginal impact of career concerns (whatever they may be) on teamwork. The marginal impact of each agent's career concerns on his teammate's direct career effect is as follows (i,j=1,2, $i \neq j$):

$$\frac{\partial^2 \tau_i^X}{\partial k_i \partial k_j} = -\frac{s}{32} \frac{\left(k_i^2 + 3(3k_j - m)(k_j + m) - 2k_i(11k_j + m)\right)}{k_i^{1/4} k_j^{3/4} (k_1 + k_2 + m)^3}$$

For a sufficiently junior agent ($k_i > 3k_j$), increases in his teammate's career concerns lessen his direct career effect. A junior agent chooses a lower level of teamwork as his own career concerns increase and a higher level of teamwork as his teammate's career concerns increase. Taken together, these marginal effects overlap as a result of the interdependency. For a given level of precision, the reverse is true for a sufficiently senior agent ($k_i < m < 2k_j$). For peers ($k_i = k_j$), teamwork increases in the common degree of career concerns k at a decreasing rate.

The period 1 principal's expected profit in the X regime is as follows:

$$\Pi_1^X = \frac{k_1^2(1-R_X) + k_2^2(1-R_X) + 4k_1k_2 + 2m(k_1+k_2)}{2(k_1+k_2+m)^2} + \frac{s^2(k_1k_2)^{1/2}\left[3(k_1+k_2)+4m\right]}{8(k_1+k_2+m)^2} \quad (31)$$

where $R_X \equiv r\sigma^2(k_1 + k_2 + m)$

It is clear from the above expression that expected firm profit increases at an increasing rate in the synergy parameter for all agent types and levels of precision. Note that although there are no explicit incentives because the performance measures are not contractible, the agent still bears some risk from the implicit incentives. From (31), if the risk premium R_X is too high, profit becomes negative and the principal prefers to shut down. To guarantee that expected period 1 profit is positive, assume R_X <1.[38]

The effect of agent i's career concerns on profit is as follows (i,j=1,2, $i \neq j$):

$$\frac{\partial \Pi_1^X}{\partial k_i} = \frac{k_j^{\,2} + \frac{R_X}{2}\left(k_j^{\,2} - k_i^{\,2}\right) - k_i k_j (1 + R_X) + m(2k_j - k_i R_X) + m^2}{(k_1 + k_2 + m)^3} + \frac{s^2 k_j^{\,1/2}\left[3(k_j^{\,2} - k_i^{\,2}) + m(7k_j - 3k_i + 4m)\right]}{16 k_i^{\,1/2}(k_1 + k_2 + m)^3}$$

Thus, if agent i is more senior than his teammate (k_i<k_j), profit increases in agent i's career concerns. However, for a given level of precision, profit decreases in the career concerns of a sufficiently junior agent, as demonstrated in Figure 3.1.[39] This decrease happens because teamwork decreases in the career concerns of a junior agent, which in turn lowers expected profit.

[38] The assumption that R_X <1limits the generalizability of the results to settings with either mildly risk averse agents or performance measures with low variance. The desirability of a low variance setting is one of the key takeaways of Parker *et al.* [2000], who found that incentive plans based on performance measures considered too noisy by team members were not particularly effective. As a result, the study recommends that, for best results, firms invest in providing reasonably low variance measures.
[39] The shape of the curve in Figure 3.1 implies the existence of an optimal level of career concerns. For $k_1 \neq k_2$, however, the second order condition can be positive or negative, which implies that this critical point need not be a maximum. For k_1=k_2, firm profit strictly increases in career concerns, so there is no corresponding downturn as depicted in Figure 3.1.

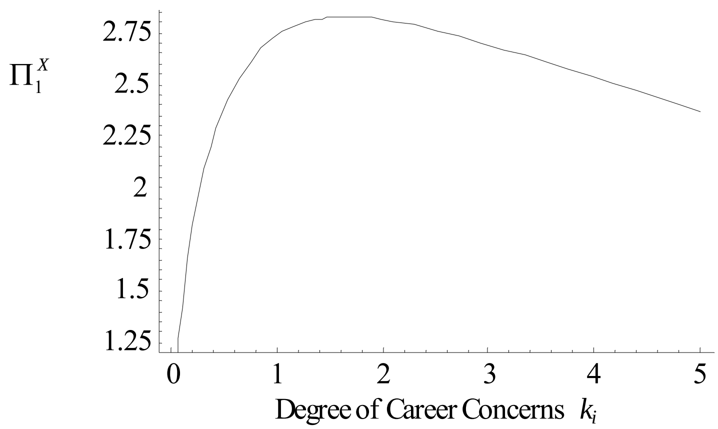


Figure 3.1. Expected firm profit in the X regime as a function of k_i. (m=1, r=.5, σ^2=.25, s=4, k_j=1)

Next, consider the impact of one agent's career concerns on his teammate's direct career effect on expected profit. This expression is given by:

$$\begin{aligned}\frac{\partial^2 \Pi_1^X}{\partial k_1 \partial k_2} &= \frac{2(k_1 k_2 - m^2) - (k_1 - k_2)^2}{(k_1 + k_2 + m)^4} + R_X \frac{m(k_1 + k_2) + 2k_1 k_2}{(k_1 + k_2 + m)^4} \\ &+ \frac{s^2}{32} \frac{m^2 (k_1 + k_2) + 4m^3}{(k_1 k_2)^{1/2} (k_1 + k_2 + m)^4} + \frac{s^2}{32} \frac{3(k_1 + k_2 + 2m)\left[4k_1 k_2 - (k_1 - k_2)^2\right]}{(k_1 k_2)^{1/2} (k_1 + k_2 + m)^4}\end{aligned} \tag{32}$$

Note that this expression decreases in team heterogeneity, $(k_i - k_j)^2$. For a sufficiently junior ($k_1k_2 > m^2$) and homogeneous team ($k_1 \to k_2$), increases in one teammate's career concerns enhance the other's direct career effect on expected profit. However, there exist values of career concerns for which teammates reduce each others' direct career impact. Figure 3.2 provides an example in which the cross-partial can be positive or negative (the horizontal line at zero is included for clarity). Note that there is a local maximum for teammates of relatively similar seniority.

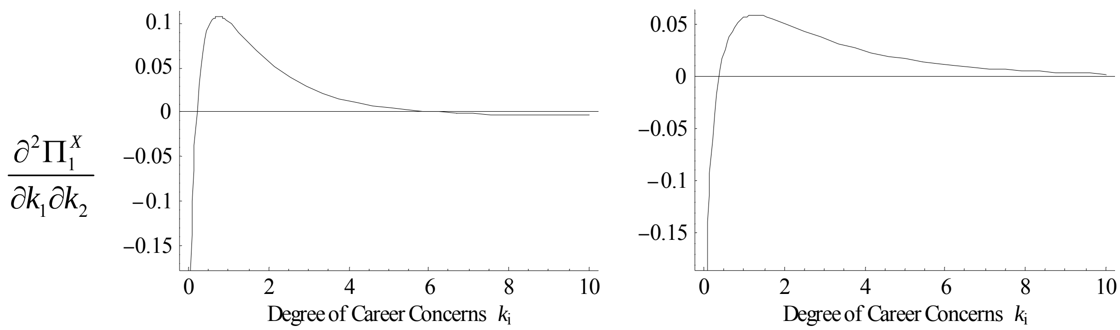


Figure 3.2. Cross-partial derivative on profit in the *X* regime with respect to career concerns. (m=1, r=.5, σ^2=.25, s=2, left panel k_j=1, right panel k_j=2)

Consider a team of two "peer" agents who have the same degree of career concerns (k_1=k_2).[40] The profit of a firms with a peer-to-peer team equals $\frac{k(3k+2m)(s^2+4)-4kR_X}{4(2k+m)^2}$, which increases in the common degree of career concerns at a decreasing rate if m>k or ms^2>4k. In this case, a more junior team – those with high career concerns – is associated with higher expected profit. Further, as synergy increases, the junior team becomes even more valuable. The above observations are summarized in the below proposition.

PROPOSITION 3.1. When team output x is the only observable performance measure and x is not contractible,

1) Synergy
   a) Individual effort is not affected by synergy, teamwork increases in synergy at a constant rate, and expected profit increases in synergy at an increasing rate.
   b) Synergy reinforces the marginal impact of career concerns (whether positive or negative) on teamwork.
2) Career concerns

[40] Note that while peers have the same degree of career concerns (i.e., the labor market's priors about their ability are equally diffuse), they are not identical *per se*: for example, they can have different realizations of and expected values of ability ($\mu_1 \neq \mu_2$).

a) Each agent's individual effort increases at a decreasing rate in his career concerns.

b) Senior agents ($k_i<k_j$) and junior agents ($k_i>3k_j+3m$):[41] A senior (junior) agent's teamwork and expected firm profit both increase (decrease) in the senior (junior) agent's career concerns, i.e., as the agent becomes less senior or more junior. A senior (junior) agent chooses less (more) teamwork as his teammate's career concerns increase.

c) Interactions between agents' career concerns

   i) A senior (junior) agent's direct career effect on individual effort, $\partial e_i/\partial k_i$, is dampened (enhanced) by increases in his teammate's career concerns, k_j. If $m<2k_j$, a senior (junior) agent i's direct career effect on teamwork, $\partial \tau_i/\partial k_i$, increases (decreases) in his teammate's career concerns.

   ii) Teammates can strengthen or lessen each others' direct career effect on expected first period profit, $\partial \Pi_1/\partial k_i$. For a sufficiently homogeneous (heterogeneous) team relative to the precision of team output, teammates strengthen (lessen) the marginal impact of each others' career concerns on expected firm profit.

3) Peer-to-peer teams

   a) Individual effort, teamwork and expected firm profit increase at a decreasing rate in the common degree of career concerns, k. Career concerns are even more valuable as synergy increases.

The main takeaways from Proposition 3.1 are as follows. First, as expected, firms prefer a higher synergy parameter, ceteris paribus. Second, for homogeneous teams, the firm is better off with junior agents. For heterogeneous teams, however, the impact of an agent's career concerns on expected firm profit and teamwork depends on the type of

[41] These conditions are sufficient but not necessary conditions.

team member. Figures 3.1 and 3.2 imply that, when only team output is observable, more homogeneous teams are associated with higher expected profit.

3.3.2 Individual performance measures observable (Regime *Y*)

In this regime, team output is not observable. Instead, the parties observe non-contractible individual performance measures for each agent. All parties update their beliefs about the agent's type based on the observed individual measures y_{11} and y_{21}. Recall that agent *j*'s measure is not informative about agent *i*'s ability or transient shock. Agent *i*'s period 2 fixed wage is as follows (*i*=1,2):

$$\alpha_{i2}^{Y} = E[a_i \mid y_{11}, y_{21}] = \mu_i + \frac{k_i}{k_i + n}(y_{i1} - \hat{y}_{i1}) \tag{33}$$

To solve for each agent's period 1 effort choices, substitute the fixed wage from (33) into the agent's certainty equivalent in (26), take first order conditions and solve.

LEMMA 3.2. When non-contractible individual input measures y_{11} and y_{21} are the only observable period 1 performance measures, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (*i*=1,2):

$$e_i^Y = \frac{k_i}{k_i + n}, \quad \tau_i^Y = f_{ii}\frac{k_i}{k_i + n} \tag{34}$$

As with the team output regime (*X*), each agent's career concerns induce both individual effort and teamwork.[42] In regime *Y*, however, the labor market observes no measures with a synergy component, so the agents do not incorporate the interdependency into their effort choices, and thus synergy has no impact on either effort level. It is straightforward to see from (34) that both of agent *i*'s effort choices increase in his own career concerns at a decreasing rate, and neither type of effort is a function of his teammate's career concerns k_j (i.e., there are direct career effects but no indirect career

[42] Note that when agent *i*'s teamwork is not reflected in his own performance measure (i.e., f_{ii}=0), he chooses no teamwork.

effects in regime *Y*). Accordingly, neither team member's career concerns strengthens or lessens his teammate's direct career effect (i.e., all cross-partials are zero).

When the only observable measures are individual input measures, expected firm profit is given by:

$$\Pi_1^Y = s\left(\frac{f_{11} f_{22} k_1 k_2}{(k_1+n)(k_2+n)}\right)^{1/2} + \sum_{i=1,2}\left(\frac{k_i n + k_i^2(1-f_{ii}^2 - R_{Yi})}{(k_i+n)^2}\right) \tag{35}$$

where $R_{Yi} = r\sigma^2(k_i+n)$

In the *Y* regime, to guarantee that expected period 1 profit is positive, assume $f_{ii}^2+R_{Yi}<1$. For $f_{ii}>0$, profit increases in synergy at a constant rate. The effect of agent *i*'s career concerns on profit is as follows (i,j=1,2, $i\neq j$):

$$\frac{\partial \Pi_1^Y}{\partial k_i} = \frac{s}{2}\frac{k_j^{1/2}}{k_i^{1/2}}\frac{n(f_{11}f_{22})^{1/2}}{(k_i+n)^{3/2}(k_j+n)^{1/2}} + \frac{n\left(n-k_i(f_{ii}^2+R_{Yi})\right)-k_i^2 R_{Yi}/2}{(k_i+n)^3}$$

Note that expected firm profit may increase or decrease in career concerns, as demonstrated in Figure 3.3. If agent *i* is sufficiently senior (i.e., $k_i<n/(f_{ii}^2+R_{Yi})$ or synergy is sufficiently high $s > \frac{2k_i^{3/2}+k_i^2 R_{Yi}}{(f_{11}f_{22})^{1/2}(k_i+n)^{3/2}}$, profit increases in agent *i*'s career concerns.

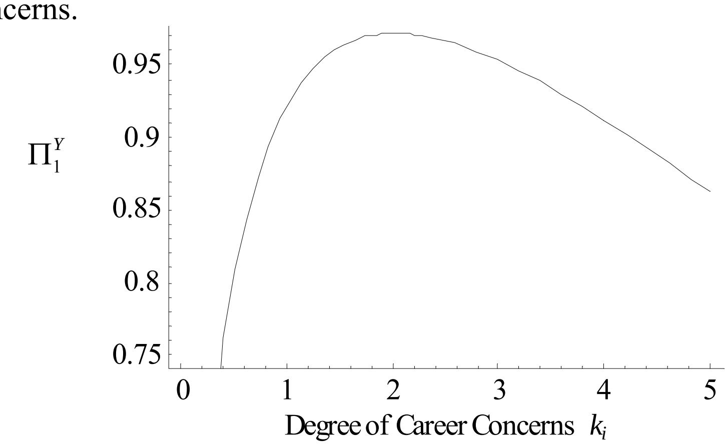


Figure 3.3. Expected firm profit in the *Y* regime. (f_{ii}=.25, f_{jj}=.25, k_j=1, n=1, r=.5, σ^2=.25, s=2)

Next, consider the impact of one agent's career concerns on his teammate's direct career effect on expected profit:

$$\frac{\partial^2 \Pi_1^Y}{\partial k_1 \partial k_2} = \frac{sn}{2}\left(\frac{f_{11} f_{22}}{k_1 k_2 \left(k_1+n\right)^3 \left(k_2+n\right)^3}\right)^{1/2}$$

The cross partial is strictly positive, which indicates that for all agent types, an agent's direct career effect on profit (whether positive or negative) is reinforced by his teammate's career concerns. Thus, in regime *Y*, a junior agent is a more desirable teammate for an agent whose career concerns increase profit, whereas a senior agent is a more desirable teammate for an agent whose career concerns decrease profit.

Finally, consider the interaction between synergy and career concerns. From (35) it is clear that the cross-partial is positive. Thus, synergy is more valuable to the principal in the presence of higher career concerns (more junior agents).

When agents are peers ($k_i = k_j = k$ and $f_{11} = f_{22} = f_{ii}$), expected profit equals the following:

$$\Pi_1^Y = k\frac{k(1-f_{ii}^2 - R_Y) + 2n + sf_{ii}(k+n)}{(k+n)^2}$$

As in the *X* regime, this expression is not monotonic in career concerns for all parameter values because $\frac{\partial \Pi_1^Y}{\partial k} = \frac{n(2n + sf_{ii}(k+n) - 2f_{ii}^2 k) - R_Y k(k+2n)}{\left(k+n\right)^3}$. Here, a homogeneous senior team can be preferred to a heterogeneous team with higher career concerns when the synergy parameter is sufficiently low. For example, (k_1, k_2) = (1.75,1.75) is preferred to (1.75, 2.0) in Figure 3.4 below.

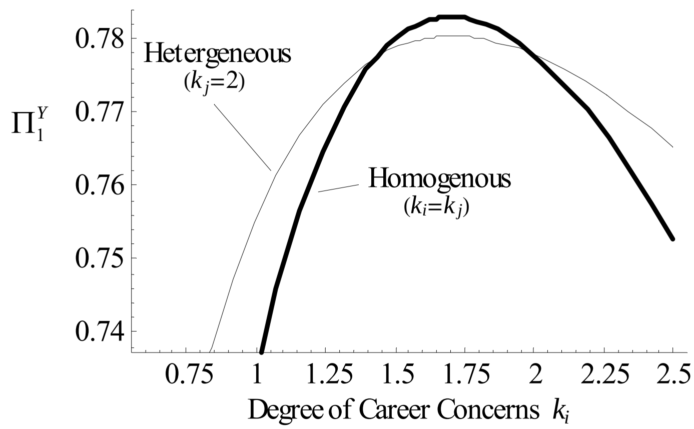


Figure 3.4. Expected firm profit in the *Y* regime. ($f_{11}= f_{22}$ =.25, *n*=1, *r*=.5, σ^2=.25, *s*=1/2)

The above observations are summarized in the below proposition.

PROPOSITION 3.2. When individual input measures y_{11} and y_{21} are the only observable performance measures and these measures are not contractible,

1) Synergy
   a) The synergy parameter has no impact on either effort level. Expected profit increases at a constant rate in synergy.
   b) Synergy increases the marginal impact of career concerns (whether positive or negative) on expected profit.
2) Career concerns
   a) Individual effort and teamwork increase at a decreasing rate in an agent's own career concerns; his teammate's degree of career concerns has no impact on the agent's effort choices.
   b) Expected profit increases in career concerns for a sufficiently senior team member ($k_i<n/(f_{ii}^2+ R_{Yi})$) or for sufficiently high synergy. Regardless of type, each agent's career concerns strengthens the marginal impact (positive or negative) of his teammate's career concerns.

3) Peer-to-peer teams

   a) A homogeneous team with lower career concerns can be preferred to a heterogeneous team with higher career concerns (see Figure 3.4).

The main takeaways of Proposition 3.2 are as follows. First, as in the *X* regime, the period 1 principal strictly prefers more synergy, ceteris paribus. Second, in a high-synergy environment, the firm is strictly better off with junior team members; however, in contrast to the *X* regime, in a low-synergy setting, this need not hold, even for homogeneous teams.

Compared with the team output regime (*X*), fewer results depend on the distinction between junior and senior agents in individual measure regime than in the team output regime, which is intuitively appealing because the team members do not incorporate the more complex team interdependency in their effort choices, and so the distinction is not as relevant.

3.3.3 All performance measures observable (Regime *XY*)

In this regime, both team output and individual performance measures are observable but not contractible. As before, all parties update their beliefs about the agent's type based on the observed period 1 measures. Agent *i*'s period 2 fixed wage is as follows:

$$\alpha_{i2}^{XY} = E[a_i \mid x_1, y_{11}, y_{21}] = \mu_i + \rho_{ix}(x_1 - \hat{x}_1) + \rho_{iyi}(y_{i1} - \hat{y}_{i1}) + \rho_{iyj}(y_{j1} - \hat{y}_{j1}) \tag{36}$$

where:

$$\left.\begin{aligned} \rho_{ix} &\equiv \text{corr}(a_i, x_1 \mid y_{11}, y_{21}) = \frac{1}{D} k_i n(k_j + n) \\ \rho_{iyi} &\equiv \text{corr}(a_i, y_{i1} \mid x_1, y_{j1}) = \frac{1}{D}\left[k_i mn + k_1 k_2 (m+n)\right] \\ \rho_{iyj} &\equiv \text{corr}(a_i, y_{j1} \mid x_1, y_{i1}) = -\frac{1}{D} k_1 k_2 n \end{aligned}\right\} \text{ for } i, j = 1, 2, \ i \neq j$$

and $D \equiv k_1 k_2 m + n\left[2k_1 k_2 + m(k_1 + k_2)\right] + n^2(k_1 + k_2 + m)$

Paralleling the *X* regime, the correlation between agent *i*'s ability and the team output (conditional on the two individual measures), ρ_{ix}, is increasing in k_i and decreasing in k_j. Interestingly, the correlation between agent *i*'s ability and his own performance measure (conditional on team output and his teammate's measure), ρ_{iyi} , increases in *both* agents' degree of career concerns, k_i and k_j. Finally, the correlation between agent *i*'s ability and his teammate's individual measure (conditional on team output and his own individual measure) decreases in both k_i and k_j.

To solve for each agent's period 1 effort choices, substitute (36) into (26), and take first order conditions. Without further assumptions, the first order conditions for teamwork are not simultaneously solvable in closed form. There are two options to overcome this tractability problem: imposing *ex ante* symmetry across agents (i.e., $k_1=k_2=k$), or removing the impact of teamwork on the individual measures (i.e., $f_{11}=f_{21}=f_{12}=f_{22}=0$). To preserve the difference among agents, and hence the differential career concerns, I proceed with the latter assumption.[43] Solving yields Lemma 3.3.

LEMMA 3.3. When team output *x* and individual input measures *y* are observable but not contractible and $f_{ii}=f_{ij}=0$, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (i,j=1,2, $i\neq j$):

$$e_i^{XY} = \rho_{ix} + \rho_{iyi}, \quad \tau_i^{XY} = \frac{s}{2}\rho_{ix}^{3/4}\rho_{jx}^{1/4} \tag{37}$$

In contrast to the *Y* regime, which has no teamwork unless the individual performance measures are sensitive to teamwork ($f_{ii}>0$), the *XY* regime has positive teamwork even when $f_{ii}=0$. This positive teamwork parallels the *X* regime, where each agent's teamwork incorporates both direct and indirect career effects.

[43] Assuming symmetry yields $\tau_{i1}^{XY} = s + f(\rho_{iyi} + \rho_{iyj}) = s + \dfrac{fkm(n+k)}{k^2 m + 2kn(k+m) + n^2(2k+m)}$.

Because ρ_{ix} and ρ_{iyi} are both increasing in agent i's career concerns, individual effort plainly increases in a team member's own career concerns. On the other hand, because ρ_{ix} decreases and ρ_{iyi} increases in agent j's career concerns, the impact of the indirect career effect on teamwork is not so clear. Agent i's individual effort is affected by his teammate's career concerns as follows:

$$\frac{\partial e_i^{XY}}{\partial k_j} = -\frac{k_i n^4}{D^2},$$

$$\text{where } D \equiv k_1 k_2 m + n\left(2k_1 k_2 + m(k_1 + k_2)\right) + n^2 (k_1 + k_2 + m)$$

Therefore, the indirect career effect on individual effort is negative, as in the X regime. Now consider the impact of the indirect career effect on the direct career effect for agent i (i,j=1,2, $i \neq j$):

$$\frac{\partial^2 e_i^{XY}}{\partial k_1 \partial k_2} = \frac{n^4}{D^3}\left[(k_i - k_j)mn + (k_i - k_j)n^2 + (k_1 k_2 - n^2)m + 2k_1 k_2 n)\right]$$

The cross-partial is more positive (or less negative) for the junior team member ($k_i > k_j$). Further, if he is sufficiently junior ($k_i > k_j n^2$) relative to the precision of the individual measures, then a junior team member's direct career effect is reinforced by the career concerns of his more senior teammate.

As in the X regime, teamwork increases in synergy at a constant rate (see (37)), and the relationship between teamwork and career concerns is more intricate than that for individual effort. Consider the direct career effect on teamwork (i,j=1,2, $i \neq j$):

$$\frac{\partial \tau_i^{XY}}{\partial k_i} = \frac{s n^2 k_j^{1/4} (k_j + n)^{3/4}}{8 D^2 k_i^{1/4} \left(k_i + n\right)^{3/4}} \left(k_1 k_2 (3m + 2n) + 3mn(k_1 + k_2) + n^2 (3(k_j + m) - k_i)\right)$$

The direct career effect on teamwork is positive for a sufficiently senior agent ($k_i < 3(k_j + m)$) or a sufficiently precise individual measure ($n < 3m$). Equivalently, having a sufficiently junior teammate (or sufficiently noisy team output) causes a senior agent's teamwork choice to increase in his own career concerns. This result parallels the *X* regime for a similarly senior agent ($k_i<k_j<3(k_j+m)$).

The indirect career effect on agent *i*'s teamwork is as follows ($i,j=1,2,\ i\neq j$):

$$\frac{\partial \tau_i^{XY}}{\partial k_j}=\frac{sn^2 k_i^{3/4}(k_i+n)^{1/4}}{8D^2 k_j^{3/4}\left(k_j+n\right)^{1/4}}\left(k_1k_2(m-2n)+mn(k_1+k_2)+n^2(k_i-3k_j+m)\right)$$

For a sufficiently imprecise team output measure ($m>2n$ and $m>3k_j-k_i$), the indirect career effect on teamwork is positive: each team member intensifies his teamwork as his teammate's career concerns increase.

The full expression for the effect of the interaction of teammates' career concerns on agent *i*'s teamwork, $\frac{\partial^2 \tau_i^{XY}}{\partial k_i k_j}$, is lengthy and presented in Appendix B. For most parameter values, this expression is positive (i.e., teammates reinforce each other's direct career effect on teamwork). However, Figure 3.5 demonstrates that this cross-partial can be positive or negative (the horizontal line at zero is included for clarity). Specifically, agent *j*'s career concerns reduce the direct career effect on agent *i*'s teamwork for sufficiently imprecise individual performance measures relative to team output measure (high *n* relative to *m*) and sufficiently junior teammate *j* relative to agent *i* (high k_j relative to k_i).

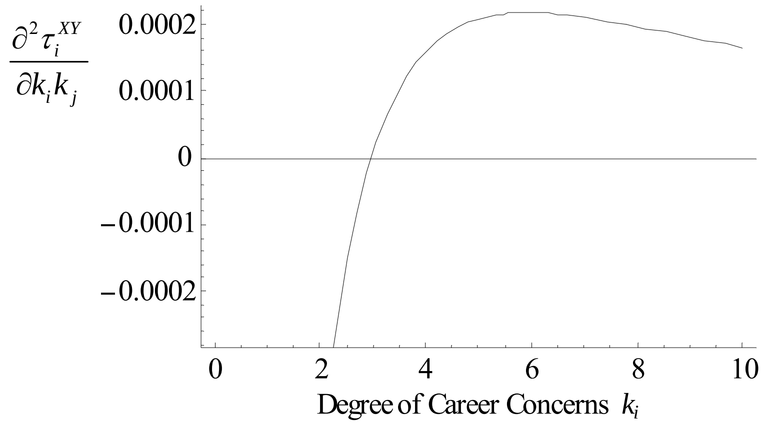


Figure 3.5. Cross-partial derivative on teamwork in the *XY* regime with respect to each agent's career concerns. (k_j=10, m=1, n=10, s=2)

When team output and individual input measures are both observable, expected firm profit in period 1 is as follows:

$$\Pi_1^{XY} = \sum_{i=1,2}\left(\rho_{ix} + \rho_{iyi} - \frac{1}{2}\left(\rho_{ix} + \rho_{iyi}\right)^2 - \frac{R_{XYi}}{2}\right) + \frac{s^2}{2}\left(\rho_{1x}\rho_{2x}\right)^{1/2}\left(1 - \frac{\rho_{1x} + \rho_{2x}}{4}\right)$$
$$\text{where } R_{XYi} \equiv R_X\,\rho_{ix}^{\;2} + R_{Yi}\rho_{iyi}^{\;2} + 2k_i\rho_{ix}\rho_{iyi} \qquad (38)$$

Expected firm profit increases at an increasing rate in the synergy parameter for all agents and levels of precision.[44] Interestingly, the indirect career effect on teamwork results above $\left(\frac{\partial \tau_i^{XY}}{\partial k_j}\right)$ in the *XY* regime suggest that for some parameter values, expected net profit may *increase* in the variance related to the shock terms, a result that runs counter to the standard LEN literature but is consistent with empirical evidence documented by Keating [1997]. In fact, this is the case: while profit strictly decreases in the variance of the shock term in team output (m), there do exist parameter values for which profit increases in the variance of the shock terms in the individual performance measures (n). Figure 3.6 presents one such example.

[44] To see this, note that $\rho_{1x} + \rho_{2x} < 1$.

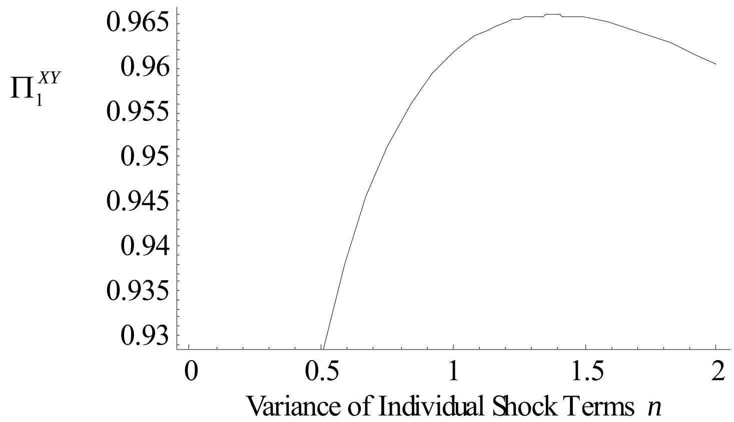


Figure 3.6. Expected firm profit in the X*Y* regime. (f_{11}=f_{22}=0,s=2,k_1=3,k_2=1, m=3, r=.5, σ^2=.25)

The marginal impact of agent i's career concerns on profit $\left(\frac{\partial \Pi_1^{XY}}{\partial k_i}\right)$ is lengthy and presented in Appendix B. Figure 3.7 illustrates that expected profit can increase or decrease in agent i's degree of career concerns; profit decreases in k_i for sufficiently junior agent i relative to his teammate j and the precision of team output. Note the contrast to regime X, where the maximum occurred for approximately equal levels of career concerns across the team.

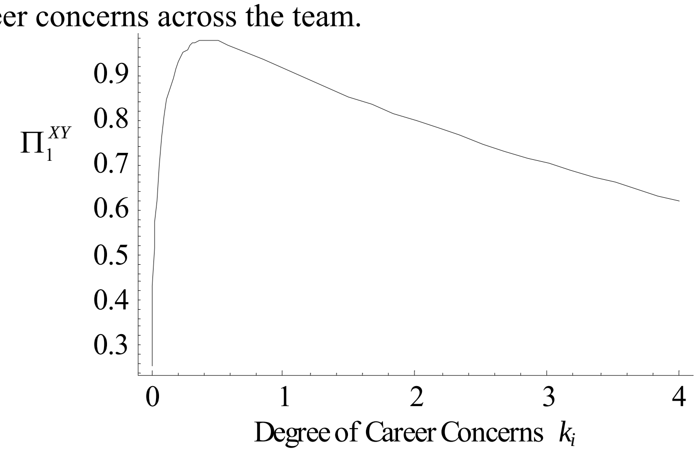


Figure 3.7. Expected firm profit in the X*Y* regime. (f_{11}=f_{22}=0, k_j=.1,m=.25,n=4, r=.5, σ^2=.25, s=2)

Finally, consider the interaction of the team members' career concerns, $\frac{\partial^2 \Pi_1^{XY}}{\partial k_1 k_2}$.

Again, the expression is lengthy and presented in Appendix B. Figure 3.8 shows that there exist parameter values for which the impact of one agent's career concerns on his teammate's direct career effect on expected profit can be positive or negative (the horizontal zero line is included for clarity).

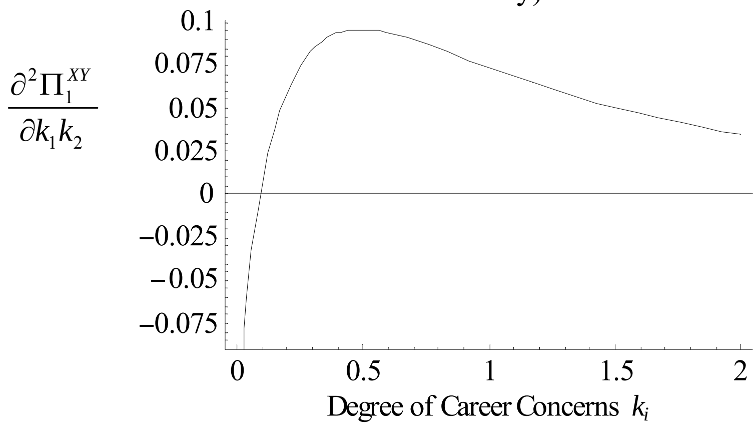


Figure 3.8. Expected firm profit in the X*Y* regime. (f_{11}=f_{22}=0, k_j=1, *m*=1, *n*=4, *r*=.5, σ^2=.25, *s*=2)

For peer-to-peer teams (k_i=k_j), individual effort, teamwork and expected profit all increase at a decreasing rate in the common degree of career concerns. The interaction between career concerns and synergy is also strictly positive, indicating that not only are junior agents more desirable, but they are even more valuable to the firm as synergy increases.

The above observations are summarized in the below proposition.

PROPOSITION 3.3. When team output and individual input measures are both observable, but none of these measures are contractible, and f_{ii}=f_{ij}=0,[45]

1) Synergy. As in regime *X* where only team output is observable,

[45] The conditions in this proposition are sufficient but not necessary conditions.

a) Individual effort is not affected by synergy, teamwork increases in synergy at a constant rate, and expected profit increases in synergy at an increasing rate.

b) Synergy reinforces the marginal impact of career concerns (whether positive or negative) on teamwork.

2) Career concerns

a) Each agent's individual effort increases at a decreasing rate in his career concerns and decreases at an increasing rate in his teammate's career concerns.

b) Each agent's teamwork increases in his own career concerns if $k_i<3k_j+3m$ (e.g., in a peer-to-peer team). The change in an agent's teamwork as his teammate's career concerns increase depends primarily on the precision of team output.

c) For most combinations of parameter values, expected firm profit increases in career concerns, but there exist parameter values for which profit decreases in career concerns (see Figure 3.7).

d) Interactions between agents' career concerns

i) A junior agent's ($k_i>k_j$) direct career effect on individual effort, $\partial e_i/\partial k_i$, is more enhanced/ less diminished by increases in his teammate's career concerns, k_j, than vice versa. With sufficiently precise (imprecise) individual measures, a junior (senior) agent's direct career effect strictly increases (decreases) in his teammate's career concerns.

ii) For most combinations of parameter values, teammates reinforce each other's direct career effect on teamwork, $\partial \tau_i/\partial k_i$. A senior agent's teamwork can decrease in his teammate's career concerns when individual measures are imprecise relative to the team output measure (high n relative to m) and his teammate is sufficiently junior.

iii) There exist parameters for which teammates can strengthen or lessen each others' direct career effect on expected first period profit, $\partial\Pi_1/\partial k_i$ (see Figure 3.8).

3) Precision

   a) For a sufficiently imprecise team output measure (m>2n and m>3 $k_j - k_i$), agent i's teamwork increases in both his own and his teammate's career concerns.

   b) Expected firm profit strictly increases in the precision of team output. Profit can either increase *or* decrease in the precision of the individual measures.

4) Peer-to-peer teams. As in regimes X and Y, where not all measures are observable,

   a) Individual effort, teamwork and expected firm profit increase at a decreasing rate in the common degree of career concerns, k. Career concerns are even more valuable as synergy increases.

The main takeaways from Proposition 3.3 are as follows. First, consistent with the other regimes, firms prefer a higher synergy parameter, ceteris paribus. Second, the team output and individual performance measures' precisions play a more important role than in the other regimes: more precision in the team output measure (or less precision in the individual measures) emphasizes the role of synergy in the agent's effort choices, whereas more precision in the individual measures (or less precision in the team output measure) reduces the role of synergy.

3.3.4 Comparison across regimes

Comparing the period 1 principal's expected profit across regimes is equivalent to comparing agency welfare. Each agent's expected overall compensation (regardless of regime) is simply twice his *ex ante* expected ability, or 2 μ_i, because the period 1 principal sets the fixed wage to exactly satisfy the participation constraints. The period 2 principal's expected profit equals zero.

Because the model has no explicit cost for each performance measure, one might expect that more performance measures are inherently better (i.e. lead to higher profit). Surprisingly, this is not the case. Consider the following example with peer-to-peer teams, where agents have the same degree of career concerns (k_1=k_2=k):

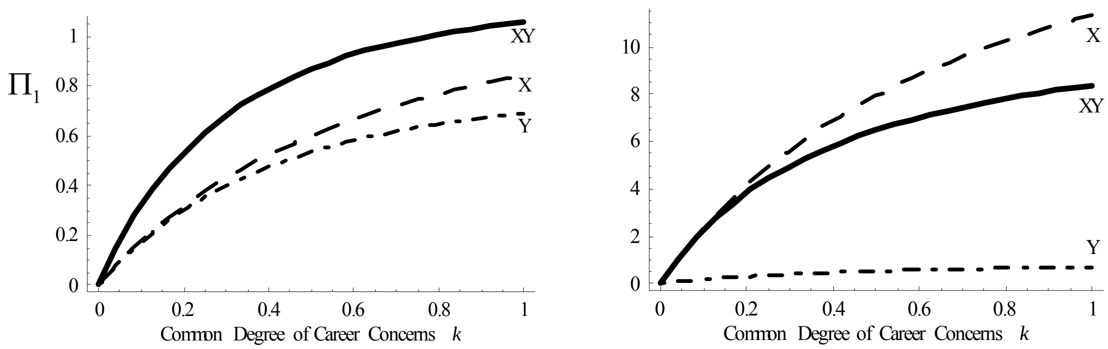


Figure 3.9. Expected firm profit in the *X*, *Y* and *XY* regimes. (f_{11}= f_{22} =0, m=1, n=1, r=.5, σ^2=.25, left panel: s=2, right panel: s=10)

Figure 3.9 illustrates that for a high synergy parameter, the team-output-only regime (*X*) can dominate the regime with incremental individual performance measures (*XY*). Both of these regimes increase in synergy at an increasing rate, whereas the individual-performance-measure-only regime (*Y*) increases in synergy at a constant rate. Thus, the addition of the individual performance measures to the team output measure dilutes the rate at which firm's expected profit increases in synergy.

It is also straightforward that either regime *X* or regime *Y* can dominate when the precision of their associated performance measure is sufficiently high relative to the other regime. However, even for comparable precision levels, either regime can dominate, as seen in Figure 3.10. For sufficiently low levels of synergy, regime *Y* has higher expected profit than regime *X*.

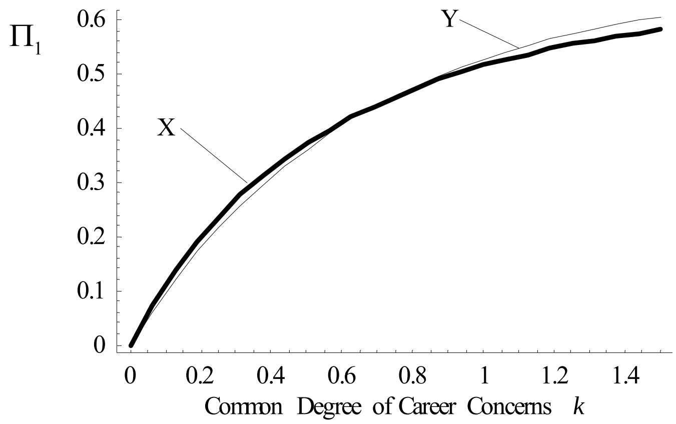


Figure 3.10. Expected firm profit in the *X* and *Y* regimes. ($f_{11}= f_{22}=0$, m=1.5, n=1.5, r=.5, σ^2=.25,s=1)

The above discussion highlights that there is a nontrivial interaction between synergy and career concerns, even for peer-to-peer teams, for whom expected profit strictly increases in career concerns. To the extent that a firm could choose which performance measures are observable, the regime under which a principal is best off depends critically on the level of potential synergy and the career concerns of team members.

3.4. CONCLUSION

This chapter investigates the interaction between team synergy and career concerns across settings where different performance measures are observed. Specifically, I study the impact on a team member's non-collaborative and collaborative effort choices and on the resulting firm profit of (i) the degree of potential synergy, (ii) a team member's own career concerns, (iii) his teammate's career concerns, (iv) the interaction between the career concerns of different team members, and (v) the interaction between synergy and career concerns.

The results indicate that even in the absence of contractible performance measures, career concerns induce positive effort levels, and the potential for synergy interacts with career concerns to produce collaborative effort. In a high synergy environment, firms are better off having at least one performance measure that reflects potential synergy, or team members under-invest in teamwork. Conversely, in a low synergy environment, firms are better off having at least one performance measure that does *not* reflect potential synergy, or team members over-invest in teamwork. Further, even in the absence of explicit costs for each performance measure, more performance measures are not necessarily better.

A regime in which only output measures (i.e., measures that reflect synergy) are observable has the highest profit in high-synergy settings. In this regime, firms are best off with a homogeneous team of junior agents (those with higher career concerns). When the team is not homogeneous, team composition has a major impact on results, with opposite results for junior and senior agents. Expected firm profit increases in a senior agent's career concerns but decreases in a junior agent's career concerns. The increase or decrease in profit is driven by a corresponding change in teamwork: senior agents choose more teamwork as career concerns increase, whereas junior agents choose less teamwork as career concerns increase.

A regime in which only individual input measures (i.e. measures that don't reflect synergy) are observable has the highest profit in low-synergy settings. In this regime, firms may be better off with junior or senior agents, depending on the extent of the potential synergy.

A regime in which both input and output measures are observable may or may not dominate regimes in which only one of the measures is observable. In this regime, performance measure precision plays an important role: the role of synergy is enhanced by more precision in the team output measure or less precision in the individual measures.

As with all analytic models, the modeling choices limit the generalizability of the results. Some of the more important assumptions include the common observance of all parameters by all parties (e.g., everyone knows the synergy parameter). The specific functional forms chosen also reduce the applicability of the model. The assumption of non-contractible performance measures, while a limitation, is mitigated by the analysis in chapter 4, which considers the same observability regimes with contractible measures.

Chapter 4: Team Synergy, Career Concerns and Incentive Pay

4.1. Introduction

This chapter is a direct extension of chapter 3, which models team synergy and differential career concerns in a multiple-agent, multiple-task dynamic model where an interactive synergy term is present. Chapter 3 considers non-contractible performance measures only. The purpose of this chapter is to explore the same phenomena, career concerns and team synergy, in a setting with contractible measures, and to compare the results across parallel contractible versus non-contractible regimes.

As in chapter 3, this chapter models a firm that hires two agents to work together on a one-period team project under the following (exogenously specified) performance measure regimes: (1) an output measure that includes a synergy component, (2) a pair of individual input measures with no such synergy component, or (3) both input and output measures. I similarly assume that the agents who comprise the team have varying degrees of career concerns and that the second period measures have no effort component; this latter assumption eliminates ratchet effects from the analysis and allows more parsimonious solutions.

I model a short-term contracting environment where all observable performance measures are contractible. I analyze the optimal individual effort and teamwork in each regime; however, limited analysis of incentive weights is possible without imposing the additional restriction of "peer" agents, i.e., agents with the same levels of career concerns.[46] With these further assumptions in place, I solve for incentive weights.

[46] For clarity, note that while peers have the same degree of career concerns, they are not identical *per se*: they can have different realizations of and expected values of ability.

The results indicate that firms are better off with higher synergy potential and teams of agents with low career concerns. In general, synergy reinforces the benefit from senior agents. This contrasts with non-contractible performance measurement regimes, where firms are better off with teams of junior agents, and synergy causes those junior agents to be even more valuable to the firm. The reason for this difference is that in the absence of explicit incentives, a firm must rely on career concerns to induce effort (so the more career concerns, the more effort), whereas when a firm can induce effort via explicit incentives, career concerns distort the effort level away from the second best (and the more career concerns, the more distortion).

The rest of the chapter is organized as follows. Section 4.2 sets up the model. Section 4.3 presents the model solutions. Section 4.4 compares non-contractible results from chapter 3 to the contractible results from Section 4.3. Section 4.5 discusses limitations and concludes.

4.2. THE MODEL[47]

Consider a two-period model where each period a risk-neutral principal hires two risk- and effort-averse agents for a one-period task. Each agent has ability a_i, which is constant across periods and firms.[48] In the first period, agent *i* (*i*=1,2) chooses individual non-negative effort, e_i, and teamwork, τ_i. In period 1, team output x_1 is a linearly additive function of agent abilities a_1 and a_2, individual efforts e_1 and e_2, a team synergy term, $s(\tau_1\tau_2)^{1/2}$, and a transient shock, ε_{x1}. In period 2, team output consists only of ability and a transient shock, as follows:[49]

---

[47] This model setup is identical to chapter 3 except for the wage structure, which includes incentive pay in this chapter. I repeat the setup here (verbatim) for the reader's convenience.

[48] To avoid confusion, the index *i* always refers to which agent, and the index *t* indicates which period.

[49] Period 2 team output could be $x_{i2} = a_i + \varepsilon_{xi2}$; that is, the team need not exist in period 2.

$$x_1 = a_1 + a_2 + e_1 + e_2 + s(\tau_1 \tau_2)^{1/2} + \varepsilon_{x1}$$
$$x_2 = a_1 + a_2 + \varepsilon_{x2}$$

where s>0 is an exogenous parameter known to all parties that represents the strength of the synergy. The individual performance measures, if they exist, are linearly additive functions of ability a_i, individual effort e_i, and teamwork τ_i and a transient shock, ε_{yit}, as follows:

$$\left.\begin{aligned} y_{i1} &= a_i + e_{it} + f_{ii}\tau_i + f_{ij}\tau_j + \varepsilon_{yi1} \\ y_{i2} &= a_i + \varepsilon_{yi2} \end{aligned}\right\} \quad i = 1, 2$$

This functional form reflects that disentangling agent's individual effort from his teamwork may not be possible. For example, billable hours may reflect a combination of time spent in individual effort *and* helping one's teammates. When this is the case, only an aggregate measure of the two types of effort is observable, and the parameter $f_{ii} \in [0,1]$ represents the weight of agent i's teamwork (relative to his individual effort) on his own measure. Further, a team member's individual performance measure may increase when his teammate assists him. The parameter $f_{ij} \in [0,1]$ represents the weight of agent j's teamwork (relative to agent i's individual effort) on agent i's individual performance measure.[50]

For simplicity, let the agents' abilities be independent of each other and of all transient shocks; likewise, let all shock terms be independent. The random variables are normally distributed as follows:

---

[50] I restrict the upper bound on f_{ii} and f_{ij} to ensure positive expected firm profit in all regimes (specifically, in the regime where only individual measures are observable). It seems intuitive that one or both of the f_{ii} and f_{ij} parameters will be zero in certain cases. For generality, I include both parameters whenever tractable.

$$
\begin{aligned}
&a_i \sim N(\mu_i, k_i\sigma^2), \quad \varepsilon_{xt} \sim N(0, m\sigma^2), \quad \varepsilon_{yit} \sim N(0, n\sigma^2) \\
&x_1 \sim N(\mu_1 + \mu_2 + e_1 + e_2 + s(\tau_1\tau_2)^{1/2},\ (k_1 + k_2 + m)\sigma^2) \\
&x_2 \sim N(\mu_1 + \mu_2, (k_1 + k_2 + m)\sigma^2) \\
&y_{i1} \sim N(\mu_i + e_i + f_{ii}\tau_i + f_{ij}\tau_j, (k_i + n)\sigma^2) \\
&y_{i2} \sim N(\mu_i, (k_i + n)\sigma^2)
\end{aligned}
$$

The variable μ_i represents the *ex ante* expected value of agent i's ability. The variable k_i represents the amount of variance in team output that relates to ability and can be interpreted as agent i's degree of career concerns: higher values of k_i represent more diffuse priors about the agent's ability. The variables m and n, respectively, represent the performance measure variance related to the transient shock in the team output and individual performance measures.

Each agent's cost of effort is a twice-differentiable convex increasing function of individual effort and teamwork. For simplicity, assume the costs of each type of effort are additively separable. In period 2, there is no effort and accordingly no effort cost. In period 1, total cost of effort c_i has the functional form:

$$c_i = \frac{1}{2}e_i^2 + \frac{1}{2}\tau_i^2, \quad \text{for } i = 1, 2$$

Each agent has a constant absolute risk aversion (CARA) utility function:

$$\left.\begin{aligned} u_i^N &\equiv -\exp\{-r_i[w_{i1}^N + w_{i2}^N - c_i^N]\} \\ u_{i2}^N &\equiv -\exp\{-r_i[w_{i2}^N - c_i] \mid p_1^N\} \end{aligned}\right\} i = 1, 2 \tag{39}$$

where $r_i \in (0,\infty)$ is the Arrow-Pratt measure of absolute risk aversion for agent i and p_1^N consists of the observable period 1 performance measures in regime N.[51]

For tractability, each principal offers both agents a linear wage in each period based on the performance measures contractible in the regime:

$$w_{it} = \alpha_{it} + \beta_{it}x_t + \gamma_{it}y_{it} + \delta_{it}y_{jt}, \quad \text{for } i, j, t = 1, 2,\ i \neq j$$

[51] Regime $N \in \{XC, YC, XYC\}$ refers to the contractible regime where only performance measures x or y or both, respectively, are observable.

where the incentive weight on any non-contractible performance measure is zero.

All parties – the principal and all labor market firms – have access to all observable period 1 performance measures. Conditional on observing these realizations, the principal and labor market update their priors about each agent's ability. The labor market is perfectly competitive and sets each agent's reservation utility as follows:

$$U_{i1}^{N} \equiv E[a_i] = \mu_i \text{, and } U_{i2}^{N}(p_1^N) \equiv E[a_i \mid p_1^N], \quad i = 1,2 \tag{40}$$

4.3. Analysis

In this section, I consider performance measurement regimes where exogenously specified performance measures are contractible. Obtaining closed form solutions for the incentive weights requires assuming peer agents ("symmetry"); while I do not impose symmetry to obtain effort solutions, I do impose symmetry to obtain expressions for incentive weights and expected profit.

All regimes are solved by backward induction. Because the agents have no period 2 action (and no associated effort cost), the principal offers no incentive; the incentive has a marginal cost but no marginal benefit. Thus, all period 2 incentive weights equal zero (hereafter suppressed in the notation) and the fixed wage equals the reservation wage. Further, the fixed wages in the contractible period 2 regimes are exactly equivalent to the non-contractible regimes in chapter 3, and $\alpha_{i2}^{N} = E[a_i \mid p_1^N]$, which implies:

$$\begin{aligned}
\alpha_{i2}^{XC} &= E[a_i \mid x_1] = \mu_i + \frac{k_i}{k_1 + k_2 + m}(x_1 - \hat{x}_1) \\
\alpha_{i2}^{YC} &= E[a_i \mid y_{11}, y_{21}] = \mu_i + \frac{k_i}{k_i + n}(y_{i1} - \hat{y}_{i1}) \\
\alpha_{i2}^{XYC} &= E[a_i \mid x_1, y_{11}, y_{21}] = \mu_i + \rho_{ix}(x_1 - \hat{x}_1) + \rho_{iyi}(y_{i1} - \hat{y}_{i1}) + \rho_{iyj}(y_{j1} - \hat{y}_{j1})
\end{aligned} \tag{41}$$

Substituting α_{i2}^{N} into agent i's expected utility from (39) and rewriting utility in the certainty equivalent form yields:

$$ACE_i^N \equiv \alpha_{i1}^N + \beta_i^N Ex_t + \gamma_i^N Ey_{i1} + \delta_i^N Ey_{j1} + E[a_i \mid p_1^N] - c_i^N, \quad i = 1,2 \tag{42}$$

The period 1 principal maximizes expected profit as follows:

$$\left.\begin{aligned} &\underset{\substack{\alpha_{i1},\beta_i,\gamma_i,\\ \delta_i,e_i,\tau_i}}{\text{Max}} \ \Pi_1 \equiv Ex_1 - Ew_{11} - Ew_{21} \\ &\text{subject to (PC) } Ew_{i1} + Ew_{i2} - c_i - \frac{r}{2}\text{var}(w_{i1} + w_{i2}) \geq U_{i1}^N + EU_{i2}^N \\ &\qquad\qquad \text{(IC) } e_i, \tau_i \in \arg\max Eu_i^N \end{aligned}\right\} \ i = 1,2$$

The period 1 principal's objective function can be rewritten as:

$$\Pi_1^N = Ex_1 - \sum_{i=1,2} c_i - \frac{r}{2}\text{var}(w_{i1} + w_{i2}) - \mu_i + EU_{i2}^N \tag{43}$$

4.3.1 Team output measure contractible (Regime *XC*)

In this subsection, I consider a contracting regime where team output is observable and contractible, but no individual performance measures exist (i.e., $\gamma_i = \delta_i = 0$). To solve for each agent's period 1 effort choices, substitute α_{i2}^{XC} into (42), take first order conditions and solve.

LEMMA 4.1. When contractible team output x is the only observable performance measure, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (i,j=1,2, $i \neq j$):

$$e_i^{XC} = \beta_i^{XC} + \frac{k_i}{k_1 + k_2 + m} \equiv B_i^{XC}, \quad \tau_i^{XC} = \frac{s}{2}\left(B_i^{XC}\right)^{3/4}\left(B_j^{XC}\right)^{1/4} \tag{44}$$

PROOF: All proofs are in Appendix C.[52]

Collectively, the explicit and implicit incentives represent the total incentive weight on period 1 individual effort, which I define as B_i for convenience. Agent i's teamwork equals $\frac{s}{2} e_i^{3/4} e_j^{1/4} > 0$. The synergy term induces positive teamwork that is a

[52] There is no subgame problem in this or any regime modeled. That is, neither agent has a profitable deviation when his teammate chooses the equilibrium effort level. Furthermore, although some of the regimes have multiple equilibria, there is only one equilibrium in each regime with positive effort levels: the other equilibria have zero, negative or complex effort levels.

function of both an agent's own career concerns k_i (the "direct" career effect) and those of his teammate, k_j (the "indirect" career effect).[53]

To solve for the incentive weights, impose symmetry (i.e., let $k_1=k_2=k$, $r_1=r_2=r$, $f_{11}=f_{22}=f_{ii}$, and $f_{12}=f_{21}=f_{ij}$), substitute the agent's solutions into (43), take first order conditions and solve.

LEMMA 4.2. When contractible team output x is the only observable performance measure, the incentive weight for symmetric agents is as follows (i=1,2):

$$\beta_i^{XC} = \frac{s^2+4}{s^2+4+4r\sigma^2(2k+m)} - \frac{k}{2k+m} \tag{45}$$

The explicit incentive weight consists of the total incentive weight, $B_i^{XC} = \dfrac{s^2+4}{s^2+4+4r\sigma^2(2k+m)}$, less the common career effect. This result is consistent with prior work (e.g., Gibbons and Murphy [1992], Meyer and Vickers [1997], Autrey *et al.* [2005]) in which the principal attempts to undo implicit incentives by adjusting the explicit incentives accordingly. Without enough degrees of freedom, the principal cannot undo all implicit incentives (Autrey *et al.* [2005]); that is also the case in this model because there are two types of effort to induce and only one performance measure with which to adjust effort levels.

Substituting (45) into the effort expressions in (44) yields the expressions for individual effort and teamwork (i=1,2):

$$e_i^{XC} = \frac{s^2+4}{s^2+4+4r\sigma^2(2k+m)}, \quad \tau_i^{XC} = \frac{s}{2}\frac{s^2+4}{\left(s^2+4+4r\sigma^2(2k+m)\right)} \tag{46}$$

[53] Meaningful comparative statics cannot be performed until after solutions for the incentive weights have been obtained, because the incentive weights are also a function of the model's parameters. Solving for the incentive weights in closed form requires assuming identically distributed agents. As a result, I cannot analyze the impact of one teammate's individual career concerns on his own or his teammate's effort choices.

Clearly, both types of effort strictly increase in synergy. Further, for sufficiently high (low) career concerns or sufficiently low (high) synergy, both individual effort and teamwork increase at an increasing (decreasing) rate in synergy (*i*=1,2):

$$\frac{\partial^2 e_i^{XC}}{\partial s^2} = \frac{8r\sigma^2(2k+m)\left(4+4r\sigma^2(2k+m)-3s^2\right)}{\left(s^2+4+4r\sigma^2(2k+m)\right)^3}$$

$$\frac{\partial^2 \tau_i^{XC}}{\partial s^2} = s\frac{4r\sigma^2(2k+m)\left(12+12r\sigma^2(2k+m)-s^2\right)}{\left(s^2+4+4r\sigma^2(2k+m)\right)^3}$$

It is also straightforward to see from (46) that both types of effort strictly decrease (at an increasing rate) in the agents' common degree of career concerns, *k*. This happens because the principal sets the explicit incentive to offset the implicit incentive, which increases in *k*. Thus, since the teamwork of a pair of agents with sufficiently high career concerns decreases in career concerns, ceteris paribus, one might expect less effort from a junior team because of the costly risk associated with career concerns.

The interaction between synergy and career concerns is as follows (*i*=1,2):

$$\frac{\partial^2 e_i^{XC}}{\partial s \partial k} = s\frac{16r\sigma^2\left(s^2+4-4r\sigma^2(2k+m)\right)}{\left(s^2+4+4r\sigma^2(2k+m)\right)^3}$$

$$\frac{\partial^2 \tau_i^{XC}}{\partial s \partial k} = \frac{4r\sigma^2\left(s^4-16-4r\sigma^2(2k+m)(4+3s^2)\right)}{\left(s^2+4+4r\sigma^2(2k+m)\right)^3}$$

Here, when synergy is sufficiently high, it reinforces the overall career effect, whereas if the risk premium is sufficiently high, synergy lessens the overall career effect. Thus, a junior team might exert *even* less effort in the presence of high synergy because it exacerbates the agent's risk cost. Note that this effect is stronger for teamwork than for individual effort; it takes substantially more synergy for the career effect to be reinforced. This happens because the synergy multiplier creates relatively more risk for teamwork (which creates synergy).

The period 1 principal's expected profit in the *XC* regime is as follows:

$$\Pi_1^{XC} = \frac{(s^2+4)^2}{4\left(s^2+4+4r\sigma^2(2k+m)\right)} \tag{47}$$

Expected firm profit increases at an increasing rate in the synergy parameter but *decreases* at an increasing rate in career concerns. Thus, in the *XC* regime, senior people – those with low career concerns – are associated with higher expected profit. The interaction between synergy and career concerns affects profit as follows:

$$\frac{\partial^2 \Pi_1^{XC}}{\partial s \partial k} = -s\frac{32r^2\sigma^4(2k+m)(s^2+4)}{\left(s^2+4+4r\sigma^2(2k+m)\right)^3}$$

Therefore, synergy and career concerns are countervailing forces. As a team becomes more junior, the benefit from synergy decreases. Equivalently, as synergy increases, senior agents are even more valuable to the firm.

PROPOSITION 4.1. When contractible team output *x* is the only observable performance measure and agents have the same degree of career concerns,

1. Individual effort, teamwork and expected profit all increase in synergy at a non-constant rate and decrease at an increasing rate in career concerns.
2. Synergy and career concerns are countervailing forces on expected firm profit, but they can be reinforcing or countervailing forces on individual effort and teamwork.

4.3.2 Individual performance measures contractible (Regime *YC*)

In this subsection, I consider contracting regimes where team output is not observable or contractible (and thus β_i=0). Instead, the parties have access to individual performance measures for each agent. To solve for each agent's period 1 effort choices, substitute the *YC* regime period 2 fixed wage from (41) into the agent's certainty equivalent in (42), take first order conditions and solve.

LEMMA 4.3. When contractible individual input measures y_{11} and y_{21} are the only observable period 1 performance measures, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (i,j=1,2, i≠j):

$$e_i^{YC} = \gamma_i^{YC} + \frac{k_i}{k_i + n} \equiv G_i^{YC}, \quad \tau_i^{YC} = f_{ii} G_i^{YC} + f_{ji} \delta_i^{YC} \tag{48}$$

As in the team output regime, individual effort e_i differs only by the period 1 explicit incentive. For convenience, I define the total incentive weight, G_i, on period 1 individual effort. To solve for the incentive weights, impose symmetry, substitute the agent's solutions into (43), take first order conditions and solve.

LEMMA 4.4. When contractible individual input measures y_{11} and y_{21} are the only performance measures observable in period 1, the incentive weights for symmetric agents are as follows (i,j=1,2, i≠j):

$$\begin{aligned} \gamma_i^{YC} &= \frac{1}{D_Y}\left[f_{ji}^{\,2} + r\sigma^2 (k+n)\left(1 + \frac{s}{2} f_{ii}\right)\right] - \frac{k}{k+n} \\ \delta_i^{YC} &= \frac{1}{D_Y}\left[\frac{s}{2} f_{ji}\left(1 + r\sigma^2 (k+n)\right) - f_{ii} f_{ji}\right] \\ &\text{where } D_Y \equiv f_{ji}^{\,2} + r\sigma^2 (k+n)\left[1 + f_{ii}^{\,2} + f_{ji}^{\,2} + r\sigma^2 (k+n)\right] \end{aligned} \tag{49}$$

As in the XC regime, the explicit incentive weight on an agent's own individual measure consists of the total incentive weight, G_i^{YC}, less the common career effect. Substituting (49) into (48) yields the effort expressions for regime YC (i,j=1,2, i≠j):

$$\begin{aligned} e_i^{YC} &= \frac{1}{D_Y}\left[f_{ji}^{\,2} + r\sigma^2 (k+n)\left(1 + \frac{s}{2} f_{ii}\right)\right] \\ \tau_i^{YC} &= \frac{1}{D_Y}\left[f_{ii} r\sigma^2 (k+n)\left(1 + \frac{s}{2} f_{ii}\right) + \frac{s}{2} f_{ji}^{\,2}\left(1 + r\sigma^2 (k+n)\right)\right] \end{aligned}$$

In this regime, without a synergy component in the explicit incentive, the team's effort choices will be unaffected by synergy. As a result, the principal includes the synergy parameter (linearly) in the incentive weights, and individual effort and teamwork increase in synergy at a constant rate.

An agent's effort choices are affected by career concerns as follows (i,j=1,2, $i \neq j$):

$$\frac{\partial e_i^{YC}}{\partial k} = -\frac{r\sigma^2}{D_Y{}^2}\left[f_{ii} f_{ji}{}^2 \left(f_{ii} - \frac{s}{2} \right) + \left(f_{ji}{}^2 + r\sigma^2 (k+n) \right)^2 + \frac{s}{2} f_{ii} r^2 \sigma^4 (k+n)^2 \right]$$

$$\frac{\partial \tau_i^{YC}}{\partial k} = -\frac{r\sigma^2}{D_Y{}^2}\left[f_{ji}{}^2 \left(\frac{s}{2} - f_{ii} \right) + f_{ji}{}^2 sr\sigma^2 (k+n) + r^2\sigma^4 (k+n)^2 \left(f_{ii} + \frac{s}{2}\left(f_{ii}{}^2 + f_{ji}{}^2 \right) \right) \right]$$

Individual effort decreases in the common degree of career concerns k for either a sufficiently low synergy parameter ($s<2f_{ii}$) or a sufficiently low sensitivity of one agent's individual measure to his teammate's teamwork ($f_{ji}<r\sigma^2(k+n)$).[54] In contrast, teamwork decreases in career concerns for a sufficiently high synergy parameter ($s>2f_{ii}$). Note that since $f_{ii} \in [0,1]$, a synergy parameter of 2 or more suffices.

The expression for the rate at which effort changes in career concerns is as follows (i,j=1,2, $i \neq j$):

$$\frac{\partial^2 e_i^{YC}}{\partial k^2} = \frac{r^2\sigma^4}{D_Y{}^3}\left[\begin{array}{l} 2 f_{ii}{}^4 f_{ji}{}^2 + 2\left(f_{ji}{}^2 + r\sigma^2 (k+n) \right)^3 + 2 f_{ii}{}^2 f_{ji}{}^2 \left(1 + 2 f_{ji}{}^2 + 3r\sigma^2 (k+n) \right) \\ + s f_{ii} \left(r^3 \sigma^6 (k+n)^3 - f_{ji}{}^2 \left(1 + f_{ii}{}^2 + f_{ji}{}^2 + 3r\sigma^2 (k+n) \right) \right) \end{array} \right]$$

$$\frac{\partial^2 \tau_i^{YC}}{\partial k^2} = \frac{r^2\sigma^4}{D_Y{}^3}\left[\begin{array}{l} f_{ji}{}^2 \left(s(1+f_{ii}{}^2) - 2 f_{ii}(1 + f_{ii}{}^2 + f_{ji}{}^2) \right) + 3 f_{ji}{}^2 r\sigma^2 (k+n)(s - 2 f_{ii}) \\ +3 s f_{ji}{}^2 r^2 \sigma^4 (k+n)^2 (s - 2 f_{ii}) + r^3 \sigma^6 (k+n)^3 \left(2 f_{ii} + s\left(f_{ii}{}^2 + f_{ji}{}^2 \right) \right) \end{array} \right]$$

Figure 4.1 demonstrates that these expressions can be positive or negative, depending primarily on the synergy parameter and the sensitivity of an individual measure to a teammate's teamwork (the horizontal line at zero is in included for clarity).

[54]This is so because $f_{ji} < r\sigma^2 (k+n) \Rightarrow f_{ji}{}^2 < r^2 \sigma^4 (k+n)^2 \Rightarrow \frac{s}{2} f_{ii} f_{ji}{}^2 < \frac{s}{2} f_{ii} r^2 \sigma^4 (k+n)^2$.

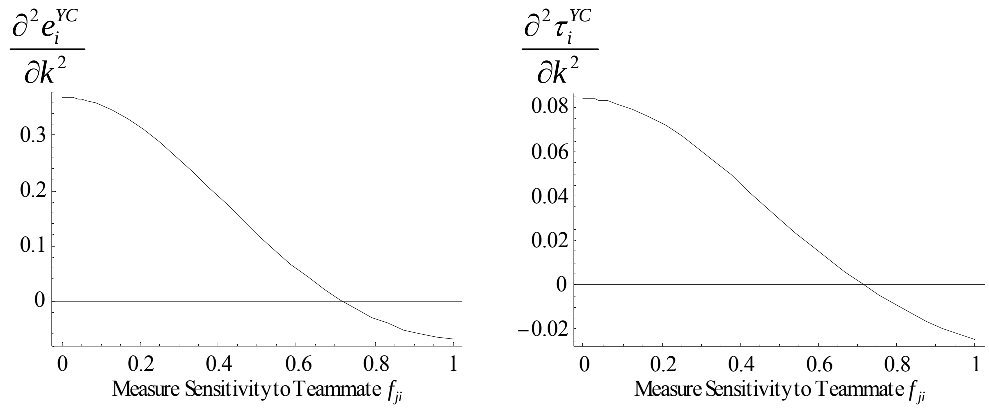


Figure 4.1. Rate of change of effort in career concerns in the *YC* regime (f_{ii}=1, n=1, r=1,σ^2=1, left panel s=9, right panel s=0.5)

The interaction between synergy and career concerns in the *YC* regime is as follows (i,j=1,2, $i\neq j$):

$$\frac{\partial^2 e_i^{YC}}{\partial s \partial k} = -\frac{r\sigma^2}{2D_Y^{\,2}}\left[f_{ii} r\sigma^2 \left(f_{ji}^{\;2} - r^2\sigma^4 (k+n)^2 \right)\right]$$

$$\frac{\partial^2 \tau_i^{YC}}{\partial s \partial k} = -\frac{r\sigma^2}{2D_Y^{\,2}}\left[f_{ji}^{\;2} \left(1 + r\sigma^2 (k+n)\right)^2 + f_{ii}^{\;2} r^2 \sigma^4 (k+n)^2 \right]$$

Thus, synergy and career concerns are strictly countervailing forces on teamwork. These forces may be complementary on individual effort if and only if ($f_{ji}<r\sigma^2(k+n)$). For example, the condition $f_{ji}<r\sigma^2(k+n)$ holds in a sufficiently noisy setting, and thus individual effort decreases (increases) as agents become more junior (senior), and synergy reinforces this reduction (expansion).

When the only observable measures are individual input measures,

$$\Pi_1^{YC} = \frac{1}{4D_Y}\left[f_{ji}^{\;2}(s^2+4) + r\sigma^2 (k+n)\left(4 + 4sf_{ii} + s^2(f_{ii}^{\;2} + f_{ji}^{\;2})\right)\right] \tag{50}$$

From this expression, it is clear that expected firm profit strictly increases at an increasing rate in the synergy parameter s. The impact of career concerns on firm profit is as follows:

$$\frac{\partial \Pi_1^{YC}}{\partial k} = -\frac{r\sigma^2}{4D_Y^{\,2}}\left[\begin{array}{l} f_{ji}^{\,2}(s-2f_{ii})^2 + 2f_{ji}^{\,2}\left(2f_{ji}^{\,2} + r\sigma^2(k+n)(s^2+4)\right)^2 \\ + f_{ii}^{\,2} r^2\sigma^4(k+n)^2\left(4+4sf_{ii} + s^2(f_{ii}^{\,2} + f_{ji}^{\,2})\right) \end{array}\right]$$

Thus, expected firm profit strictly decreases in career concerns. This is intuitively appealing in a contractible environment: the firm prefers to induce the second-best effort level, but career concerns distort the actual effort level induced. The principal doesn't have enough degrees of freedom to completely undo the career effect, and thus the more career concerns, the more distortion remains, and the lower expected profit becomes.

The impact of the interaction between synergy and career concerns on profit is:

$$\frac{\partial^2 \Pi_1^{XC}}{\partial s \partial k} = -\frac{r\sigma^2}{4D_Y^{\,2}}\left[f_{ji}^{\,2}(s-2f_{ii}) + 2sf_{ji}^{\,2} r\sigma^2(k+n) + r^2\sigma^4(k+n)^2\left(2f_{ii} + s(f_{ii}^{\,2} + f_{ji}^{\,2})\right)\right]$$

When synergy is sufficiently high relative to sensitivity (specifically, when $s>2f_{ii}$ or $s>f_{ii}/[r\sigma^2(k+n)]$), synergy reduces the rate at which profit decreases in k.[55] In general, synergy and career concerns have countervailing effects on firm profit in the *YC* regime.

Consider the special case where an agent's performance measure has no sensitivity to his teamwork, but his teammate's measure does (i.e., $f_{ii}=0$, $f_{ji}>0$). In this case, individual effort (teamwork) strictly decreases in career concerns at a non-constant (strictly decreasing) rate. Synergy has no impact on the career effect on individual effort, whereas synergy and career concerns are countervailing forces on teamwork and expected profit.

PROPOSITION 4.2. When contractible individual input measures y_{11} and y_{21} are the only performance measures observable in period 1, and agents have the same degree of career concerns,

[55] This is a sufficient, but not necessary, condition. It is difficult, but possible to find parameter values such that expected profit increases in k. One such example is $k\in[1,5], n\to 0, s\to 0$, $r = \sigma^2 = f_{ji}=1$.

1. Individual effort and teamwork increase in synergy at a constant rate; expected profit increases in synergy at an increasing rate.
2. Individual effort and teamwork can either increase or decrease in career concerns; expected profit strictly decreases in career concerns.
3. Synergy and career concerns are strictly countervailing forces on teamwork. For sufficiently high synergy ($s>2f_{ii}$), this is also true for expected firm profit. However, these forces are countervailing for individual effort if and only if ($f_{ji}>r\sigma^2(k+n)$).

In this regime, note that while either type of effort can increase in the common degree of career concerns, *k*, nevertheless, expected firm profit always decreases in career concerns. This occurs because when individual effort increases in career concerns, teamwork decreases in career concerns, and vice versa. The net effect is always that profit decreases in *k*.

4.3.3 All performance measures contractible (Regime *XYC*)

In this regime, both team output and individual performance measures are contractible. To solve for each agent's period 1 effort choices, substitute the period 2 regime *XYC* fixed wage from (41) into (42), and take first order conditions. In this regime, to obtain closed form solutions I assume $f_{11}=f_{21}=f_{12}=f_{22}=0$.

LEMMA 4.5. When contractible team output *x* and individual input measures *y* are observable, individual (non-collaborative) effort and teamwork (collaborative effort) are as follows (i,j=1,2, $i\neq j$):

$$e_i^{XYC} = B_i^{XYC} + G_i^{XYC}, \quad \tau_i^{XYC} = \frac{s}{2}\left(B_i^{XYC}\right)^{3/4}\left(B_j^{XYC}\right)^{1/4} \tag{51}$$

where $B_i^{XYC} \equiv \beta_i^{XYC} + \rho_{ix}$, $G_i^{XYC} \equiv \gamma_i^{XYC} + \rho_{iyi}$

To solve for the incentive weights, impose symmetry, substitute the agent's solutions into (43), take first order conditions and solve.

LEMMA 4.6. When team output x and individual input measures y_{11} and y_{21} are all observable and contractible in period 1, the incentive weights for symmetric agents are as follows (i=1,2):

$$\beta_i^{XYC} = \frac{1}{D_{XY}} N - \rho_{ix}$$

$$\gamma_i^{XYC} = \frac{1}{D_{XY}} \left[r\sigma^2 \left(4M - 4kn - s^2 k(k+n) \right) \right] - \rho_{iyi} \tag{52}$$

$$\delta_i^{XYC} = B_i^{XYC} \frac{k}{k+n}$$

where

$$N \equiv (k+n)\left(s^2 + r\sigma^2 (4n + s^2(k+n))\right),$$

$$M \equiv mn + k(m+2n), \text{ and}$$

$$D_{XY} \equiv N + r\sigma^2 \left(4n^2 - 4n + M\right) + 4r^2\sigma^4 (k+n)M$$

Paralleling the team-output-only and individual-measure-only regimes, the explicit incentive weight on each individual measure consists of the total incentive weight for that measure, respectively, less agent i's direct implicit incentives (if any) related to that measure.

Substituting (52) into (51) yields the effort expressions (i=1,2):

$$e_i^{XYC} = \frac{1}{D_{XY}} \left[4M + n^2 r\sigma^2 + s^2(k+n)\left(1 + r\sigma^2(k+n)\right) \right], \quad \tau_i^{XYC} = \frac{s}{2D_{XY}} N$$

Synergy influences the effort choices as follows (i=1,2):

$$\frac{\partial e_i^{XYC}}{\partial s} = \frac{8r^2\sigma^4}{D_{XY}} \left[sn(k+n)\left(mn + km + kn + r\sigma^2 (k+n)M \right) \right]$$

$$\frac{\partial \tau_i^{XYC}}{\partial s} = \frac{(k+n)}{2D_{XY}} \begin{bmatrix} 2r\sigma^2 \left(6m(k+n) + 10kn + 4n^2 + s^2(k+n)^2 \right) \\ +r^2\sigma^4 \left(16n(M+n^2) + s^2(k+n)(4L + s^2(k+n)^2) \right) \\ +4r^3\sigma^6 (k+n)M\left(4n + 3s^2(k+n)\right) + s^4(k+n) \end{bmatrix}$$

where $L \equiv 2n(3m+n) + k(6m+11n)$

Individual effort and teamwork increase in synergy at a non-constant (i.e., increasing or decreasing) rate. Because team output is observable, the agents internalize the potential synergy and incorporate it in their teamwork level, as in regime *XC*. The principal also includes the synergy parameter in the incentive weights.

The expressions for the impact of career concerns on an agent's effort are lengthy and included in Appendix C. However, these expressions are interpretable: individual effort and teamwork both strictly decrease at an increasing rate in the agents' common degree of career concerns *k*.

The interaction between synergy and career concerns in the *XYC* regime is also lengthy and included in Appendix C. Figure 4.2 illustrates that these expressions may be either positive or negative (the horizontal line at zero is included for clarity); thus, synergy and career concerns may be countervailing or reinforcing forces on effort:

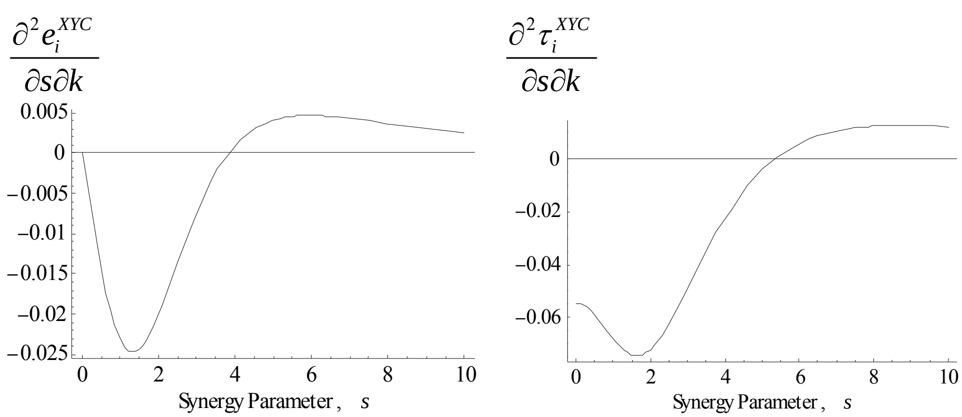


Figure 4.2. Impact of interaction between synergy and career concerns on effort, in *XYC* regime (f_{ii}= f_{ii}=0, k=1, m=1, n=4 r=σ^2=1, s=1)

In the *XYC* regime, expected firm profit is as follows (when $f_{ii} = f_{ji} = 0$):

$$\Pi_1^{XYC} = \frac{1}{D_{XY}}\left[s^2(k+n)\left(N+4+4nr\sigma^2\right)+r\sigma^2\left(16(M+n^2)\right)\right]$$

$$\text{where } N \equiv (k+n)\left(s^2+r\sigma^2(4n+s^2(k+n))\right), M \equiv mn+k(m+2n) \tag{53}$$

$$\text{and } \quad D_{XY} \equiv N+r\sigma^2\left(4n^2-4n+M\right)+4r^2\sigma^4(k+n)M$$

The comparative static expressions for the impact on expected firm profit of synergy, career concerns, and the interaction of synergy and career concerns are lengthy and included in Appendix C. However, these expressions are interpretable: profit strictly increases at an increasing rate in synergy and strictly decreases at a decreasing rate in career concerns. Furthermore, synergy and career concerns have a strictly countervailing impact on expected profit.

PROPOSITION 4.3. When team output x and individual input measures y_{11} and y_{21} are all observable and contractible in period 1, and agents have the same degree of career concerns,

1. Individual effort and teamwork increase in synergy at a non-constant rate; expected profit increases in synergy at an increasing rate.
2. Individual effort and teamwork decrease at an increasing rate in career concerns; expected profit strictly decreases at a decreasing rate in career concerns.
3. Synergy and career concerns may have a countervailing or a reinforcing impact on individual effort and teamwork. However, these forces are strictly countervailing for expected profit.

4.3.4 Comparison across regimes

In this subsection, I compare the results across regimes, focusing on the impact on total agency welfare. Note that comparing the period 1 principal's expected profit across regimes is equivalent to comparing agency welfare because each agent's expected compensation simply equals twice his *ex ante* expected ability, or 2 μ_i, regardless of regime, and the period 2 principal's expected profit equals zero.[56]

[56] As in chapter 3, the period 1 principal sets the fixed wage to exactly satisfy the participation constraints in all regimes. The participation constraint equals $U_1+E[U_2]$, and both of these terms equal μ_i.

Profit is not identical across regimes, despite the fact that in each regime the principal does exactly offset the career component in effort. Conditional on a given performance measurement regime, profit is identical whether the labor market observes the performance measure or not. However, this does not imply that scenarios with different performance measures are identical. Further, although the principal reverses the career effect on effort, career concerns remain a component of the risk premium.

First, consider the difference in profit between regime *XC* and *YC*. Both regimes increase in synergy at an increasing rate. When $f_{ii}=f_{ji}=0$, synergy favors regime *XC* more than regime *YC*, and career concerns favor regime *YC* over *XC*:

$$\Pi_1^{XC} - \Pi_1^{YC} = \frac{(s^2+4)^2}{4\left(s^2+4+4r\sigma^2(2k+m)\right)} - \frac{1}{1+r\sigma^2(k+n)}$$

When $f_{ii}=f_{ji}=1$, the difference between regimes is as follows:

$$\Pi_1^{XC} - \Pi_1^{YC} = \frac{r\sigma^2\left[(s^2+4)P + r\sigma^2(k+n)Q\right]}{4\left(s^2+4+4r\sigma^2(2k+m)\right)\left(1+r\sigma^2(k+n)(3+r\sigma^2(k+n))\right)}$$

where $\mathrm{P} \equiv sk(s-4) - 4m + n(8+s(s-4))$

$$Q \equiv n(s^2+4)^2 - 8m(2+2s+s^2) + k(s^4 - 8s^2 - 32s - 16)$$

In this case, very high synergy still favors the *XC* regime, but intermediate values of synergy may now favor the *YC* regime instead. This is demonstrated in Figure 4.3.

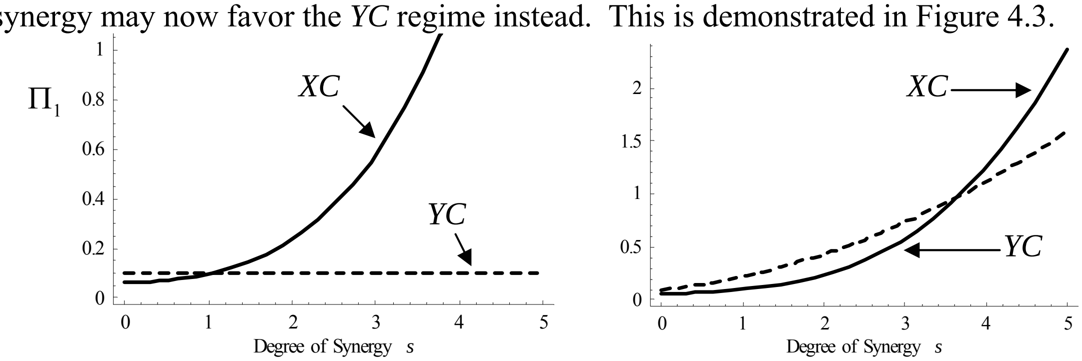


Figure 4.3. Expected firm profit in the *XC* and *YC* regimes (k=3, m=1.5, n=1.5, r=2,σ^2=1, left panel f_{ii}= f_{ii}=0, right panel f_{ii}= f_{ii}=1)

Next, I consider the difference between the *XYC* regime, in which both types of measures available, and the *XC* and *YC* regimes, in which only one type of measure is available. Because the model has no explicit cost for each performance measure, one might expect that more performance measures are inherently better (i.e. lead to higher profit). This is clearly the case in the *XYC* versus *YC* regime comparison:

$$\Pi_1^{XYC} - \Pi_1^{YC} = \frac{N}{4D_1} > 0$$

$$\text{where } D_1 \equiv 4\left[\begin{pmatrix} s^2(k+n)\left(1+r\sigma^2(k+n)\right)^2 + 2r\sigma^2(M+mn+2n^2) \\ +r^2\sigma^4(k+n)(8M+4n^2)+4r^3\sigma^6(k+n)^2 M \end{pmatrix}\right]$$

The *XYC* versus *XC* comparison, however, is ambiguous:

$$\Pi_1^{XYC} - \Pi_1^{XC} = \frac{r\sigma^2\left[(s^2+4)s^2k^2 + 2r\sigma^2(k+n)(R-4km(k+n)s^2\right]}{4D_2}$$

$$\text{where } \mathrm{D}_2 \equiv \left(s^2+4+4r\sigma^2(2k+m)\right)\left[\begin{pmatrix} s^2(k+n)+r\sigma^2(2M+2mn+4n^2+s^2(k+n)^2) \\ +4r^2\sigma^4(k+n)(mn+k(m+2n)) \end{pmatrix}\right]$$

$$\mathrm{R} \equiv 8km(k+m)+8n(2k^2+2km+m^2)+k^2s^4(k+n)$$

For sufficiently extreme (high or low) values of synergy, career concerns, or variance in team output, $R > 4km(k+n)s^2$ and *XYC* dominates *XC*.

Collectively, these comparisons across regimes provide a measure of the expected value of supplementing a contractible individual measure with a contractible team output measure.

4.4. Comparison Across Contractible and Non-Contractible Regimes

This section compares the contractible performance measure results from the analysis above with the results for non-contractible performance measures derived in chapter 3. I use the non-contractible results for identically distributed team members (the "peer-to-peer" teams special case) in this comparison because solving the contractible case in closed form requires this restriction.

Firms with exogenously contractible performance measures are better off with more senior people, whereas firms with exogenously non-contractible performance measures receive more benefit from junior people. Intuitively, in the absence of contractibility, firms must rely on career concerns to induce individual effort and synergy to induce teamwork. However, contractible performance measures provide the firm more degrees of freedom to induce desired effort levels, and thus the firm can adjust the incentive weights to reduce the agent's effort level accordingly.

These observations are summarized in the following corollary.

COROLLARY 4.1. For teams of identically distributed agents,

1. Profit increases in synergy regardless of regime or contractibility.
2. With non-contractible measures, profit increases in career concerns. With contractible measures, profit decreases in career concerns.
3. With non-contractible measures, synergy reinforces the benefit of a junior team. With contractible measures, if at least one measure is an output measure, synergy enhances the benefit of a senior team. With only a contractible individual measure, synergy may either strengthen or lessen the benefit of a senior team.

4.5. CONCLUSION

This chapter studies the role of career concerns, team synergy and the interaction between the two. I model a team setting where two agents choose individual, non-collaborative effort that does not generate synergy and collaborative effort (teamwork) that does generate synergy. The labor market can observe one or more types of contractible performance measures – either team output or individual input measures or both – which creates career concerns.

The results indicate that firms with contractible performance measures are better off with teams of senior agents. As expected, firms also prefer more synergy to less. Furthermore, when an output measure is available (regimes *XC* and *XYC*), higher synergy makes senior agents even more valuable to the firm. When only individual measures are available (regime *YC*), synergy may either reinforce or reduce the benefits of senior agents. Specifically, in a setting with low synergy, low risk premium and high sensitivity of agent's individual measure to his own teamwork, synergy diminishes the benefits of senior agents.

The model predicts that the role of career concerns depends critically on contractibility: career concerns are an essential and desirable agent characteristic in a non-contractible setting, but a dysfunctional and undesirable agent characteristic in a contractible setting. The model also provides a measure of the expected value of supplementing an existing contractible measure of one type (collective team output or individual input measure) with a contractible measure of the other type.

As with all analytic models, the modeling choices limit the generalizability of the results. Some of the more important assumptions include the common observance of all parameters by all parties (e.g., everyone knows the synergy parameter). The specific functional forms chosen also reduce the applicability of the model. The assumption of contractible performance measures, while a limitation, is mitigated by the analysis in chapter 3, which considers the same observability regimes with non-contractible measures.

Appendix A

First-best regime.

$$\Pi^{FB} = e_1 + e_2 + s(\tau_1\tau_2)^{1/2} - \frac{1}{2p_1}e_1^2 - \frac{1}{2q_1}\tau_1^2 - \frac{1}{2p_2}e_2^2 - \frac{1}{2q_2}\tau_2^2$$

$$\frac{\partial}{\partial e_i}\Pi^{FB} = 1 - e_i / p_i = 0 \Rightarrow e_i^{FB} = p_i \quad \text{for } i = 1,2$$

$$\frac{\partial}{\partial \tau_i}\Pi^{FB} = \frac{s}{2}\left(\frac{\tau_j}{\tau_i}\right)^{1/2} - \tau_i / q_i = 0 \Rightarrow \tau_j = \frac{4}{s^2 q_i^2}\tau_i^3 \Rightarrow \tau_j = \frac{4}{s^2 q_i^2}\left(\frac{4}{s^2 q_j^2}\tau_j^3\right)^3$$

$$\Rightarrow \tau_i^{FB} = \frac{s}{2} q_i^{3/4} q_j^{1/4}$$

$$\Pi^{FB} = s\left(\frac{s^2}{4}q_i q_j\right)^{1/2} + \sum_{\substack{i,j=1,2\\ i\neq j}} p_i - \frac{p_i}{2} - \frac{1}{2}\left(\frac{s^2}{4}q_i^{3/2} q_j^{1/2}\right)$$

$$\Rightarrow \; \Pi^{FB} = \frac{1}{2}\left(p_i + p_j\right) + \frac{s^2}{4}\left(q_i q_j\right)^{1/2}$$

Regime *Y*.

$$CE_i^Y = \alpha_i + \gamma_i e_i + \delta_i e_j - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\operatorname{var} w_i \Rightarrow e_i^Y = p_i \gamma_i^Y, \quad \tau_i^Y = 0, \; i = 1,2$$

$$\Pi^Y = p_i\gamma_i + p_j\gamma_j - \sum\left(\frac{1}{2}p_i\gamma_i^2 + \frac{r_i}{2}(\gamma_i^2\sigma_{yi}^2 + \delta_i^2\sigma_{yj}^2 + 2\gamma_i\delta_i\rho_y\sigma_{yi}\sigma_{yj})\right)$$

$$\frac{\partial}{\partial \delta_i}\Pi^Y = -\delta_i r_i \sigma_{yj}^2 - \gamma_i\rho_y r_i\sigma_{yi}\sigma_{yj} = 0 \Rightarrow \delta_i^Y = -\gamma_i^Y\frac{\rho_y\sigma_{yi}}{\sigma_{yj}}$$

$$\frac{\partial}{\partial \gamma_i}\Pi^Y = p_i - \gamma_i(p_i + r_i\sigma_{yi}^2) - \delta_i^Y\rho_y r_i\sigma_{yi}\sigma_{yj} = 0 \Rightarrow \gamma_i^Y = \frac{p_i}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)}$$

LEMMA 2.1. Regime *Z*.

$$CE_i^Z = \alpha_i + \kappa_i e_i + \lambda_i e_j - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\operatorname{var} w_i \Rightarrow e_i^Z = 0, \quad \tau_i^Z = q_i\kappa_i^Z$$

$$\Pi^Z = s(q_i\kappa_i q_j\kappa_j)^{1/2} - \sum\left(\frac{1}{2}q_i\kappa_i^2 + \frac{r_i}{2}(\kappa_i^2\sigma_{zi}^2 + \lambda_i^2\sigma_{zj}^2 + 2\kappa_i\lambda_i\rho_z\sigma_{zi}\sigma_{zj})\right)$$

$$\frac{\partial}{\partial\lambda_i}\Pi^Z = -\lambda_i r_i\sigma_{zj}^2 - \kappa_i\rho_z r_i\sigma_{zi}\sigma_{zj} = 0 \Rightarrow \lambda_i^Z = -\kappa_i^Z\frac{\rho_z\sigma_{zi}}{\sigma_{zj}}$$

$$\frac{\partial}{\partial\kappa_i}\Pi^Z = \frac{s}{2}\left(\frac{q_i q_j\kappa_j}{\kappa_i}\right)^{1/2} - \kappa_i(q_i + r_i\sigma_{zi}^2) - \lambda_i^Z\rho_z r_i\sigma_{zi}\sigma_{zj} = 0$$

$$\Rightarrow \kappa_i^Z = \frac{s}{2}\frac{(q_i q_j)^{1/2}}{(q_i + r_i\sigma_{zi}^2(1-\rho_z^2))^{3/4}(q_j + r_j\sigma_{zj}^2(1-\rho_z^2))^{1/4}} \quad \text{for } i, j = 1,2, i \neq j$$

▲

Conditions for regime *Y*>*Z*.

$$\Pi^Y - \Pi^Z > 0 \qquad D_Z \equiv (q_i + r_i\sigma_{zi}^2(1-\rho_z^2))^{1/2}(q_j + r_j\sigma_{zj}^2(1-\rho_z^2))^{1/2}$$

$$\Leftrightarrow \frac{1}{2}\left(\frac{p_i}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)} + \frac{p_j}{p_j + r_j\sigma_{yj}^2(1-\rho_y^2)}\right) > \frac{s^2}{4}\frac{q_i q_j}{D_Z}$$

$$\Leftrightarrow \frac{2D_Z}{q_i q_j}\left(\frac{p_i}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)} + \frac{p_j}{p_j + r_j\sigma_{yj}^2(1-\rho_y^2)}\right) > s^2$$

LEMMA 2.2. Regime *YZ*.

$$CE_i^{YZ} = \alpha_i + \gamma_i e_i + \delta_i e_j + \kappa_i e_i + \lambda_i e_j - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\operatorname{var} w_i$$

$$\Rightarrow e_i^{YZ} = p_i\gamma_i^{YZ}, \quad \tau_i^{YZ} = q_i\kappa_i^{YZ}$$

$$\begin{aligned}\Pi^{YZ} &= p_i\gamma_i + p_j\gamma_j + s(q_i q_j\kappa_i\kappa_j)^{1/2} \\ &\quad -\sum\begin{pmatrix}\frac{1}{2}p_i\gamma_i^2 + \frac{1}{2}q_i\kappa_i^2 + \frac{r_i}{2}(\gamma_i^2\sigma_{yi}^2 + \delta_i^2\sigma_{yj}^2 + 2\gamma_i\delta_i\rho_y\sigma_{yi}\sigma_{yj}) \\ +\frac{r_i}{2}(\kappa_i^2\sigma_{zi}^2 + \lambda_i^2\sigma_{zj}^2 + 2\kappa_i\lambda_i\rho_z\sigma_{zi}\sigma_{zj})\end{pmatrix} \\ &= \Pi^Y + \Pi^Z \Rightarrow \gamma_i^{YZ} = \gamma_i^Y,\ \delta_i^{YZ} = \delta_i^Y,\ \kappa_i^{YZ} = \kappa_i^Z,\ \lambda_i^{YZ} = \lambda_i^Z, \quad \text{for } i = 1,2, i \neq j\end{aligned}$$

▲

LEMMA 2.3. Regime X.[57]

$$CE_i^X = \alpha_i + \beta_i(e_i + e_j + s(\tau_i\tau_j)^{1/2}) - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\operatorname{var} w_i \Rightarrow e_i^X = p_i\beta_i^X$$

$$\frac{\partial}{\partial\tau_i}CE_i^X = \frac{\beta_i s}{2}\left(\frac{\tau_j}{\tau_i}\right)^{1/2} - q_i\tau_i = 0 \Rightarrow \tau_j = \frac{4\tau_i^3}{s^2 q_i^2\beta_i^2} \quad \Rightarrow \tau_i^X = \frac{s}{2}\left(q_i\beta_i^X\right)^{3/4}\left(q_j\beta_j^X\right)^{1/4}$$

$$\Pi^X = p_i\beta_i + p_j\beta_j + s\left(\frac{s^2}{4}q_i\beta_i q_j\beta_j\right)^{1/2} - \sum\left(\frac{1}{2}p_i\beta_i^2 + \frac{r_i}{2}\beta_i^2\sigma_x^2 + \frac{1}{2q_i}\left(\frac{s^2}{4}(q_i\beta_i)^3(q_j\beta_j)\right)^{1/2}\right)$$

$$\frac{\partial}{\partial\beta_i}\Pi^X = p_i - \beta_i(p_i + r_i\sigma_x^2) + \frac{s^2}{16}\left(\frac{q_i q_j\beta_j}{\beta_i}\right)^{1/2}(4 - q_j\beta_j - 3q_i\beta_i) = 0, \quad \text{for } i = 1,2, i \neq j$$

▲

The sign of $\partial e_i^X/\partial\beta_j^X$.

From equation (12), $e_i^X = p_i\,\beta_i^X$. Thus, $p_i(\partial\beta_i^X/\partial\beta_j^X) = \partial e_i^X/\partial\beta_j^X$. Define equation (14) as $G(\beta_i, \beta_j)=0$. Per the Implicit Function Theorem,

If (β_i^*, β_j^*) solves $G(\beta_i, \beta_j)=0$ and if $\frac{\partial G}{\partial\beta_i}(\beta_i^*, \beta_j^*) \neq 0$, then $\exists$ a C^1 function $\beta_i(\beta_j)$

near β_j^* such that $\beta_i(\beta_j^*) = \beta_i^*$ and $\frac{\partial\beta_i}{\partial\beta_j}(\beta_j^*) = -\frac{\frac{\partial G}{\partial\beta_j}(\beta_i^*, \beta_j^*)}{\frac{\partial G}{\partial\beta_i}(\beta_i^*, \beta_j^*)}$. Thus,

$$\frac{\partial\beta_i}{\partial\beta_j}(\beta_j^*) = -\frac{\frac{\partial G}{\partial\beta_j}(\beta_i^*, \beta_j^*)}{\frac{\partial G}{\partial\beta_i}(\beta_i^*, \beta_j^*)} = -\frac{\frac{s^2}{32}\left(\frac{q_i q_j}{\beta_i^*\beta_j^*}\right)^{1/2}(4 - 3q_i\beta_i^* - 3q_j\beta_j^*)}{-\left(p_i + r_i\sigma_x^2\right) - \frac{s^2}{32}\left(\frac{q_i q_j\beta_j^*}{(\beta_i^*)^3}\right)^{1/2}(4 + 3q_i\beta_i^* - q_j\beta_j^*)}$$

$$\Rightarrow \frac{\partial e_i}{\partial\beta_j}(\beta_j^*) = \frac{p_i\frac{s^2}{32}\left(\frac{q_i q_j}{\beta_i^*\beta_j^*}\right)^{1/2}(4 - 3q_i\beta_i^* - 3q_j\beta_j^*)}{\left(p_i + r_i\sigma_x^2\right) + \frac{s^2}{32}\left(\frac{q_i q_j\beta_j^*}{(\beta_i^*)^3}\right)^{1/2}(4 + 3q_i\beta_i^* - q_j\beta_j^*)}$$

---

[57] Second order conditions $\frac{\partial^2}{\partial\beta_i^2}\Pi^X = -p_i - r_i\sigma_x^2 - \frac{s^2}{32}\left(\frac{q_i q_j\beta_j}{\beta_i^3}\right)^{1/2}(4 + 3\beta_i - \beta_j) < 0,$ because β_j <4.

This expression can be positive or negative. Two examples (which differ only in q):

1) If s=2, r_i=2, r_j=1, σ_x=1, p_i=p_j=1, q_i=q_j=1, then (β_i^*, β_j^*)=(0.506604, 0.663951) solves G=0, $\partial G/\partial\beta_i(\beta_i^*, \beta_j^*)\neq 0$, and $\partial\beta_i/\partial\beta_j(\beta_i^*, \beta_j^*) = \partial e_i/\partial\beta_j$= 0.0240758.

2) If s=2, r_i=2, r_j=1, σ_x=1, p_i=p_j=1, q_i=q_j=4, then (β_i^*, β_j^*)=(0.148744, 0.621495) solves G=0, $\partial G/\partial\beta_i(\beta_i^*, \beta_j^*)\neq 0$, and $\partial\beta_i/\partial\beta_j(\beta_i^*, \beta_j^*) = \partial e_i/\partial\beta_j$= -0.335905.

Regime X for identical agents.

Substituting $p_i=p_j=p$ and $q_i=q_j=q$ and $r_i=r_j=r$ into equation (14) yields:

$$\frac{\partial}{\partial\beta_i}\Pi^X = p-\beta(p+r\sigma_x{}^2)+\frac{s^2q}{4}(1-q\beta)=0 \;\Rightarrow \beta_i^X = \frac{4p+s^2q}{4p+s^2q+4r\sigma_x{}^2}$$

The resulting profit increases in s^2 at an increasing rate:

$$\frac{\partial}{\partial s^2}\Pi^X = \frac{q(s^2q+4p)(s^2q+4p+8r\sigma_x{}^2)}{8(s^2q+4(p+r\sigma_x{}^2))^2} > 0, \quad \frac{\partial^2}{\partial(s^2)^2}\Pi^X = \frac{8q^2r^2\sigma_x{}^4}{(s^2q+4(p+r\sigma_x{}^2))^3} > 0$$

Conditions for regime $X>Y$.

$$\Pi^X > \Pi^Y \qquad [\Sigma_y{}^2 \equiv \sigma_y{}^2(1-\rho_y{}^2)]$$

$$\Leftrightarrow \frac{(s^2q+4p)^2}{4(s^2q+4(p+r\sigma_x{}^2))} > \frac{p}{p+r\Sigma_y{}^2}$$

$$\Leftrightarrow (s^2q+4p)^2(p+r\Sigma_y{}^2) > 4p(s^2q+4(p+r\sigma_x{}^2))$$

$$\Leftrightarrow s^4pq^2+8s^2p^2q+16p^3+(s^4q^2+8s^2pq+16p^2)r\Sigma_y{}^2$$

$$> 4s^2pq+16p^2+16pr\sigma_x{}^2$$

$$\Leftrightarrow (4p+s^2q)\big((4p+s^2q)(p+r\Sigma_y{}^2)-4p\big) > 16pr\sigma_x{}^2$$

PROPOSITION 2.1. Conditions for regime $X>Z$.

$$\Pi^X > \Pi^Z \qquad [\Sigma_z{}^2 \equiv \sigma_z{}^2(1-\rho_z{}^2),\;\; D_Z \equiv q+r\Sigma_z{}^2]$$

$$\Leftrightarrow \frac{(s^2q+4p)^2}{4(s^2q+4(p+r\sigma_x{}^2))} > \frac{s^2}{4}\frac{q^2}{D_Z}$$

$$\Leftrightarrow (s^2q+4p)^2D_Z > s^2(s^2q^3+4q^2(p+r\sigma_x{}^2))$$

$$\Leftrightarrow (s^4q^2+8s^2pq+16p^2)D_Z > s^4q^3+4s^2q^2(p+r\sigma_x{}^2)$$

$$\Leftrightarrow s^4q^2(D_Z-q)+4s^2pq(2D_Z-q)+16p^2D_Z > 4s^2q^2r\sigma_x{}^2$$

$$\Leftrightarrow s^2r\Sigma_z{}^2+4\frac{p}{q}(2r\Sigma_z{}^2+q)+\frac{16p^2}{s^2q^2}(q+r\Sigma_z{}^2) > 4r\sigma_x{}^2$$

Regime X or Z favored by q.

$$\frac{\partial}{\partial q}(\Pi^X - \Pi^Z > 0) \Leftrightarrow \frac{s^2}{4}\frac{(s^2q+4p)^2 + 8r\sigma_x{}^2(s^2q+4p)}{(s^2q+4(p+r\sigma_x{}^2))^2} > \frac{s^2}{4}\frac{q(q+2r\Sigma_z{}^2)}{(q+r\Sigma_z{}^2)^2}$$

$$\Leftrightarrow [(s^2q+4p)^2 + 8r\sigma_x{}^2(s^2q+4p)](q+r\Sigma_z{}^2)^2 > q(q+2r\Sigma_z{}^2)(s^2q+4(p+r\sigma_x{}^2))^2$$

$$\Leftrightarrow \left(r\Sigma_z{}^2(s^2q+4p) - 4qr\sigma_x{}^2\right)\left(r\Sigma_z{}^2(s^2q+4p) + 4r\sigma_x{}^2(q+2r\Sigma_z{}^2)\right) > 0$$

$$\Leftrightarrow r\Sigma_z{}^2(s^2q+4p) - 4qr\sigma_x{}^2 > 0$$

$$\Leftrightarrow s^2qr\Sigma_z{}^2 > 4qr\sigma_x{}^2 - 4pr\Sigma_z{}^2$$

$$\Leftrightarrow s^2 > \frac{4q\sigma_x{}^2 - 4p\Sigma_z{}^2}{q\Sigma_z{}^2} = 4\frac{\sigma_x{}^2}{\Sigma_z{}^2} - \frac{p}{q} \quad \blacktriangle$$

Conditions for regime $X>YZ$.

$\Pi^X > \Pi^{YZ}$: [Define $\Sigma_y{}^2 \equiv \sigma_y{}^2(1-\rho_y{}^2)$, $\Sigma_z{}^2 \equiv \sigma_z{}^2(1-\rho_z{}^2)$.]

$$\Leftrightarrow \frac{(s^2q+4p)^2}{4(s^2q+4(p+r\sigma_x{}^2))} > \frac{p}{p+r\Sigma_y{}^2} + \frac{s^2}{4}\frac{q^2}{(q+r\Sigma_z{}^2)}$$

$$\frac{\partial}{\partial s^2}\left(\Pi^X - \Pi^{YZ}\right) = \frac{q[(8p+2s^2q)(s^2q+4p+4r\sigma_x{}^2) - (s^2q+4p)^2]}{4(s^2q+4p+4r\sigma_x{}^2)^2} - \frac{q^2}{4(q+r\Sigma_z{}^2)} > 0$$

$$\Leftrightarrow 4q[8r^2\sigma_x{}^2\left(\Sigma_z{}^2(s^2q+4p) - 2\sigma_x{}^2\right) + r\Sigma_z{}^2(s^2q+4p)^2] > 0$$

$$\therefore \frac{s^2q+4p}{2} > \frac{\Sigma_z{}^{-2}}{\sigma_x{}^{-2}} \Rightarrow \frac{\partial}{\partial s^2}\left(\Pi^X - \Pi^{YZ}\right) > 0$$

$$\frac{\partial}{\partial p}\left(\Pi^X - \Pi^{YZ}\right) = \frac{(s^2q+4p)(s^2q+4(p+2r\sigma_x{}^2))}{(s^2q+4(p+r\sigma_x{}^2))^2} - \frac{r\Sigma_y{}^2}{(p+r\Sigma_y{}^2)^2}$$

$$\frac{\partial}{\partial q}\left(\Pi^X - \Pi^{YZ}\right) = \frac{s^2\left(r\Sigma_z{}^2(s^2q+4p) - 4qr\sigma_x{}^2\right)\left(r\Sigma_z{}^2(s^2q+4p) + 4r\sigma_x{}^2(q+2r\Sigma_z{}^2)\right)}{4(q+r\Sigma_z{}^2)^2(s^2q+4(p+r\sigma_x{}^2))^2}$$

$$\therefore \frac{s^2}{4} + \frac{p}{q} > \frac{\Sigma_z{}^{-2}}{\sigma_x{}^{-2}} \Rightarrow \frac{\partial}{\partial q}\left(\Pi^X - \Pi^{YZ}\right) > 0$$

LEMMA 2.4. Regime *XY*.

$$CE_i^{XY} = \alpha_i + \beta_i(e_i + e_j + s(\tau_i\tau_j)^{1/2}) + \gamma_i e_i + \delta_i e_j - \frac{1}{2p_i}e_i^2 - \frac{1}{2q_i}\tau_i^2 - \frac{r_i}{2}\operatorname{var} w_i$$

$$\Rightarrow e_i^{XY} = p_i(\beta_i^{XY} + \gamma_i^{XY}), \tau_i^{XY} = \frac{s}{2}\left(q_i\beta_i^{XY}\right)^{3/4}\left(q_j\beta_j^{XY}\right)^{1/4}$$

$$\Pi^{XY} = s\left(\frac{s^2}{4}q_i q_j \beta_i \beta_j\right)^{1/2} + \sum\begin{pmatrix} p_i(\beta_i + \gamma_i) - \frac{1}{2}p_i(\beta_i + \gamma_i)^2 - \frac{1}{2q_i}\left(\frac{s^2}{4}(q_i\beta_i)^{3/2}(q_j\beta_j)^{1/2}\right) \\ -\frac{r_i}{2}(\beta_i^2\sigma_x^2 + \gamma_i^2\sigma_{yi}^2 + \delta_i^2\sigma_{yj}^2 + 2\gamma_i\delta_i\rho_y\sigma_{yi}\sigma_{yj}) \end{pmatrix}$$

$$\frac{\partial}{\partial\delta_i}\Pi^{XY} = -\delta_i r_i \sigma_{yj}^2 - \gamma_i\rho_y r_i\sigma_{yi}\sigma_{yj} = 0 \Rightarrow \delta_i^{XY} = -\gamma_i^{XY}\frac{\rho_y\sigma_{yi}}{\sigma_{yj}}$$

$$\frac{\partial}{\partial\gamma_i}\Pi^{XY} = p_i - p_i\beta_i - \gamma_i(p_i + r_i\sigma_{yi}^2) - \delta_i\rho_y r_i\sigma_{yi}\sigma_{yj} = 0 \Rightarrow \gamma_i^{XY} = \frac{p_i(1-\beta_i^{XY})}{p_i + r_i\sigma_{yi}^2(1-\rho_y^2)}$$

$$\frac{\partial}{\partial\beta_i}\Pi^{XY} = p_i - \beta_i(p_i + r_i\sigma_x^2) + \frac{s^2}{16}\left(\frac{q_i q_j\beta_j}{\beta_i}\right)^{1/2}(4 - q_j\beta_j - 3q_i\beta_i) - \gamma_i = 0 \qquad \blacktriangle$$

PROPOSITION 2.2. Conditions for regime *XY*>*YZ*.

$$\Pi^{XY} > \Pi^{YZ}: \quad \Sigma_y^2 \equiv \sigma_y^2(1-\rho_y^2), D_Y \equiv p + r\Sigma_y^2,\ \Sigma_z^2 \equiv \sigma_z^2(1-\rho_z^2), D_Z \equiv q + r\Sigma_z^2$$

$$\Leftrightarrow \frac{1}{4}\frac{16p^2r\sigma_x^2 + (4p+s^2q)(pqs^2 + r\Sigma_y^2(4p+s^2q))}{p(s^2q + 4r\sigma_x^2) + r\Sigma_y^2(4p + s^2q + 4r\sigma_x^2)} > \frac{p}{D_Y} + \frac{s^2}{4}\frac{1}{D_Z}$$

$$\Leftrightarrow \frac{16p^2r\sigma_x^2 + (4p+s^2q)(pqs^2 + r\Sigma_y^2(4p+s^2q))}{p(s^2q + 4r\sigma_x^2) + r\Sigma_y^2(4p + s^2q + 4r\sigma_x^2)} > \frac{4(4pD_Z + s^2D_Y)}{D_Y D_Z}$$

$$\Leftrightarrow [16p^2r\sigma_x^2 + (4p+s^2q)(pqs^2 + r\Sigma_y^2(4p+s^2q))]D_Y D_Z$$
$$> 4(4pD_Z + s^2D_Y)[p(s^2q + 4r\sigma_x^2) + r\Sigma_y^2(4p + s^2q + 4r\sigma_x^2)]$$

Rearranging terms yields:

$$\Leftrightarrow (4p + s^2q)\left((4pD_Y D_Z + s^2qD_Y(D_Z - q)\right) > 4r\sigma_x^2\left(2pD_Z + s^2q^2D_Y\right) + 2pD_Z(4p + s^2q)$$

$$\Leftrightarrow (4p + s^2q)\left(4pD_Y D_Z + s^2qD_Y r\Sigma_z^2\right) > 2pD_Z(4r\sigma_x^2 + 4p + s^2q) + 4s^2q^2D_Y r\sigma_x^2 \qquad \blacktriangle$$

Appendix B

Some useful calculations follow.

Covariances.

$\operatorname{cov}(x_1, x_2) = \operatorname{var} a_1 + \operatorname{var} a_2 = (k_1 + k_2)\sigma^2$, $\operatorname{cov}(x_t, y_{it}) = \operatorname{cov}(a_i, x_t) = \operatorname{var} a_i = k_i\sigma^2$

$\operatorname{cov}(y_{i1}, y_{i2}) = \operatorname{cov}(a_i, y_{it}) = \operatorname{var} a_i = k_i\sigma^2$, $\operatorname{cov}(y_{it}, y_{jt}) = \operatorname{cov}(a_i, y_{jt}) = 0$

Variances.

$$\begin{aligned}
\operatorname{var}(w_{i1}^N + w_{i2}^N) &= \operatorname{var} w_{i1}^N + \operatorname{var} w_{i2}^N + 2\operatorname{cov}(w_{i1}^N, w_{i2}^N) \\
\operatorname{var}(w_{i1}^X + w_{i2}^X) &= \left(B_{i1}^X\right)^2 \operatorname{var} x \\
\operatorname{var}(w_{i1}^Y + w_{i2}^Y) &= \left(G_{i1}^Y\right)^2 \operatorname{var} y_i \\
\operatorname{var}(w_{i1}^{XY} + w_{i2}^{XY}) &= \left(G_{i1}^{XY}\right)^2 (k_i + n)\sigma^2 + \left(B_{i1}^{XY}\right)^2 (k_1 + k_2 + m)\sigma^2 + \left(2G_{i1}^{XY} B_{i1}^{XY}\right) k_i\sigma^2
\end{aligned}$$

$$\text{where } B_i^X = \frac{k_i}{k_1 + k_2 + m}, B_i^{XY} = \rho_{ix}, G_{i1}^Y = \frac{k_i}{k_i + n}, G_{i1}^{XY} = \rho_{iyi} \ \ (\rho\text{'s defined below})$$

Expected value.

$$E[a_i \mid x_1] = \mu_i + \frac{k_i}{k_i + k_j + m}(x_1 - \hat{x}_1)$$

$$E[a_i \mid y_{11}, y_{21}] = \mu_i + \frac{k_i}{k_i + n}(y_{i1} - \hat{y}_{i1})$$

$$E[a_i \mid x_1, y_{11}, y_{21}] = \mu_i + \rho_{ix}(x_1 - \hat{x}_1) + \rho_{iyi}(y_{i1} - \hat{y}_{i1}) + \rho_{iyj}(y_{j1} - \hat{y}_{j1})$$

$$\text{where } \rho_{ix} \equiv \frac{k_i n(k_j + n)}{D}, \rho_{iyi} \equiv \frac{k_i mn + k_1 k_2 (m + n)}{D}, \rho_{iyj} \equiv -\frac{k_1 k_2 n}{D}, \text{ and}$$

$$D \equiv k_1 k_2 m + n[2k_1 k_2 + m(k_1 + k_2)] + n^2 (k_1 + k_2 + m)$$

Proof of Lemma 3.1.

$$ACE_i^X = 2\mu_i + \frac{k_i}{k_1 + k_2 + m}(x_1 - \hat{x}_1) - \frac{1}{2}\left(e_i^X\right)^2 - \frac{1}{2}\left(\tau_i^X\right)^2$$

$$\frac{\partial ACE_i^X}{\partial e_i^X} = \frac{k_i}{k_1 + k_2 + m} - e_i^X = 0 \Rightarrow e_i^X = \frac{k_i}{k_1 + k_2 + m}$$

$$\frac{\partial ACE_i^X}{\partial \tau_i^X} = \left(\frac{s}{2}\right)\left(\frac{\tau_j^X}{\tau_i^X}\right)^{1/2} \frac{k_i}{k_1 + k_2 + m} - \tau_i^X = 0 \Rightarrow \tau_i^X = \left(\frac{s}{2}\right)\left(\frac{k_i}{k_1 + k_2 + m}\right)^{3/4} \left(\frac{k_j}{k_1 + k_2 + m}\right)^{1/4}$$

▲

Proof of Propositions 3.1, 3.2 and 3.3.

All propositions follow directly from the derivations and discussion in the text.

Proof of Lemma 3.2.

$$ACE_i^Y = 2\mu_i + \frac{k_i}{k_i+n}(y_{i1} - \hat{y}_{i1}) - \frac{1}{2}\left(e_i^Y\right)^2 - \frac{1}{2}\left(\tau_i^Y\right)^2$$

$$\frac{\partial ACE_i^Y}{\partial e_i^Y} = \frac{k_i}{k_i+n} - e_i^Y = 0 \Rightarrow e_i^Y = \frac{k_i}{k_i+n}$$

$$\frac{\partial ACE_i^Y}{\partial \tau_i^Y} = \frac{f_{ii}k_i}{k_i+n} - \tau_i^Y = 0 \Rightarrow \tau_i^Y = \frac{f_{ii}k_i}{k_i+n} \quad \blacktriangle$$

Proof of Lemma 3.3.

$$ACE_i^{XY} = 2\mu_i + \rho_{ix}(x_1 - \hat{x}_1) + \rho_{iyi}(y_{i1} - \hat{y}_{i1}) + \rho_{iyj}(y_{j1} - \hat{y}_{j1}) - \frac{1}{2}\left(e_i^{XY}\right)^2 - \frac{1}{2}\left(\tau_i^{XY}\right)^2$$

$$\frac{\partial ACE_i^{XY}}{\partial e_i^Y} = \rho_{ix} + \rho_{iyi} - e_i^{XY} = 0 \Rightarrow e_i^{XY} = \rho_{ix} + \rho_{iyi}$$

$$\frac{\partial ACE_i^{XY}}{\partial \tau_i^{XY}} = \frac{s}{2}\left(\frac{\tau_j^{XY}}{\tau_i^{XY}}\right)^{1/2} \rho_{ix} + f_{ii}\rho_{iyi} + f_{ji}\rho_{iyj} - \tau_i^{XY} = 0$$

$$\text{Let } f_{ii} = f_{ij} = 0, \text{ for } i, j = 1,2 \quad \Rightarrow \tau_i^{XY} = \frac{s}{2}\rho_{ix}^{\ 3/4}\rho_{jx}^{\ 1/4} \quad \blacktriangle$$

Lengthy expressions omitted from text:

$$\begin{aligned}\frac{\partial^2 \tau_i^{XY}}{\partial k_i k_j} = & -\Big(n^3\left(-3\,\mathrm{k1}^2\,\mathrm{k2}^2\,\mathrm{m}^2 + 2\,\mathrm{k1}\,\mathrm{k2}\,\mathrm{m}(2\,\mathrm{k1}\,\mathrm{k2} - 3\,(\mathrm{k1}+\mathrm{k2})\,\mathrm{m})\,\mathrm{n} - \right.\\
& \left(12\,\mathrm{k1}^2\,\mathrm{k2}^2 - 2\,\mathrm{k1}\,\mathrm{k2}\,(\mathrm{k1}+5\,\mathrm{k2})\,\mathrm{m} + 3\left(\mathrm{k1}^2 + 4\,\mathrm{k1}\,\mathrm{k2} + \mathrm{k2}^2\right)\mathrm{m}^2\right)\mathrm{n}^2 - \\
& 2\left(2\,\mathrm{k1}\,\mathrm{k2}\,(5\,\mathrm{k1}+\mathrm{k2}) + \left(\mathrm{k1}^2 - 4\,\mathrm{k1}\,\mathrm{k2} - 3\,\mathrm{k2}^2\right)\mathrm{m} + 3\,(\mathrm{k1}+\mathrm{k2})\,\mathrm{m}^2\right)\mathrm{n}^3 + \\
& \left.\left(\mathrm{k1}^2 + 3\,(3\,\mathrm{k2}-\mathrm{m})\,(\mathrm{k2}+\mathrm{m}) - 2\,\mathrm{k1}\,(11\,\mathrm{k2}+\mathrm{m})\right)\mathrm{n}^4\right)\mathrm{s}\Big) \Big/ \\
& \Big(32\,\mathrm{k1}^{1/4}\,\mathrm{k2}^{3/4}\,(\mathrm{k1}+\mathrm{n})^{3/4}\,(\mathrm{k2}+\mathrm{n})^{1/4} \\
& \left(\mathrm{k1}\,\mathrm{k2}\,\mathrm{m} + (2\,\mathrm{k1}\,\mathrm{k2} + (\mathrm{k1}+\mathrm{k2})\,\mathrm{m})\,\mathrm{n} + (\mathrm{k1}+\mathrm{k2}+\mathrm{m})\,\mathrm{n}^2\right)^3\Big)\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \Pi_1^{XY}}{\partial k_i} = \Bigg(& 16\,n^2 \left(m^2 n^3 (m+n) + k2^3 (m+n)^2 (m+2n) + k2^2 n (m+n) \left(3m^2 + 6mn + n^2\right) + \right. \\
& \quad \left. k2\, n^2 \left(3m^3 + m(-k1+6m)\,n + (-k1+m)\,n^2\right)\right) + \\
& \frac{1}{\sqrt{k1}\,\sqrt{k1+n}} \\
& \Big(\sqrt{k2}\; n^2 \sqrt{k2+n} \\
& \quad \Big(k1^2 \left(4k2^2 m^2 + 4k2\,m(k2+2m)\,n + m(k2+4m)\,n^2 - 3(2k2+m)\,n^3 - 3n^4\right) + \\
& \qquad n^2 (4k2\,m + 3k2\,n + 4mn)(mn + k2(m+n)) + \\
& \qquad k1\,n \left(m(8m-3n)\,n^2 + 8k2\,mn(2m+n) + k2^2 \left(8m^2 + 11mn + 6n^2\right)\right)\Big)\, s^2\Big) - \\
& 8 \\
& \Big(-k2^2 n^5 (mn + k2(m+n)) + \\
& \quad 3k1^2 n (mn + k2(m+n)) \\
& \qquad \left(n^2 (m+n)^2 + 2k2\,n(m+n)(m+2n) + k2^2 \left(m^2 + 4mn + 5n^2\right)\right) + \\
& \quad k1^3 (n(m+n) + k2(m+2n)) \\
& \qquad \left(n^2 (m+n)^2 + 2k2\,n(m+n)(m+2n) + k2^2 \left(m^2 + 4mn + 5n^2\right)\right) + \\
& \quad k1\,n^2 \left(2k2^4 n(m+n) + 2m^2 n^3 (m+n) + 2k2\,mn^2 \left(3m^2 + 6mn + 2n^2\right) + \right. \\
& \qquad \left. k2^2 n \left(6m^3 + 18m^2 n + 17mn^2 + n^3\right) + k2^3 \left(2m^3 + 8m^2 n + 15mn^2 + 6n^3\right)\right)\Big) \\
& r\sigma^2 \Bigg) \Bigg/ \\
& \left(16\left(k1\,k2\,m + (2k1\,k2 + (k1+k2)\,m)\,n + (k1+k2+m)\,n^2\right)^3\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_1^{XY}}{\partial k_1 k_2} = \Big(&n^3 \\
&\Big(\big(4\,k1^3\,k2^3\,m^3 + 2\,k1^2\,k2^2\,m^2\,(k1\,k2 + 6\,(k1 + k2)\,m)\,n + \\
&\quad k1\,k2\,m\big(24\,k1^2\,k2^2 + 5\,k1\,k2\,(k1 + k2)\,m + 12\big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m^2\big)\,n^2 + \\
&\quad 4\big(6\,k1^3\,k2^3 + 12\,k1^2\,k2^2\,(k1 + k2)\,m + k1\,k2\big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m^2 + \\
&\qquad (k1 + k2)\,\big(k1^2 + 8\,k1\,k2 + k2^2\big)\,m^3\big)\,n^3 + \\
&\quad \big(36\,k1^2\,k2^2\,(k1 + k2) + 18\,k1\,k2\big(k1^2 + 6\,k1\,k2 + k2^2\big)\,m + \\
&\qquad (k1 + k2)\,\big(k1^2 + 8\,k1\,k2 + k2^2\big)\,m^2 + 12\big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m^3\big)\,n^4 + \\
&\quad 2\big(3\,k1\,k2\big(k1^2 + 10\,k1\,k2 + k2^2\big) - 3\,(k1 + k2)\,\big(k1^2 - 10\,k1\,k2 + k2^2\big)\,m + \\
&\qquad \big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m^2 + 6\,(k1 + k2)\,m^3\big)\,n^5 - \\
&\quad \big(3\,(k1 + k2)\,\big(k1^2 - 6\,k1\,k2 + k2^2\big) + 6\big(k1^2 - 6\,k1\,k2 + k2^2\big)\,m - \\
&\qquad (k1 + k2)\,m^2 - 4\,m^3\big)\,n^6\big)\,s^2 - \\
&32\,\sqrt{k1}\,\sqrt{k2}\,\sqrt{k1 + n}\,\sqrt{k2 + n} \\
&\quad \Big(n^2\big(n\big(2\,m^2\,n^2 + k2^2\,(m + n)^2\big) + k1^2\,(m + n)\,(n\,(m + n) - 2\,k2\,(m + 2\,n)) - \\
&\qquad 2\,k1\,k2\big(k2\,(m + n)\,(m + 2\,n) + 2\,n\big(2\,m^2 + 4\,m\,n + n^2\big)\big)\big) + \\
&\quad \big(k1^2\,k2^2\,\big(k1^2 + k2^2\big)\,m^2 + \\
&\qquad k1\,k2\,m\big(3\,k1\,k2\big(k1^2 + k2^2\big) + (k1 + k2)\,\big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m\big)\,n + \\
&\qquad 2\,k1\,k2\big(k1\,k2\big(k1^2 + k2^2\big) + (k1 + k2)\,\big(k1^2 + 3\,k1\,k2 + k2^2\big)\,m + \\
&\qquad\quad 2\,(k1 + k2)^2\,m^2\big)\,n^2 + \\
&\qquad k1\,k2\,(k1 + k2)\,\big(k1^2 + (k2 + 2\,m)^2 + k1\,(3\,k2 + 4\,m)\big)\,n^3 + \\
&\qquad \big(2\,k1^2\,k2^2 - 2\,k1\,k2\,(k1 + k2)\,m - \big(k1^2 + k2^2\big)\,m^2\big)\,n^4 - \\
&\qquad (k1 + k2 + m)\,(2\,k1\,k2 + (k1 + k2)\,m)\,n^5\big)\,r\,\sigma^2\Big)\Big)\Big) \Big/ \\
&\quad \Big(32\,\sqrt{k1}\,\sqrt{k2}\,\sqrt{k1 + n}\,\sqrt{k2 + n} \\
&\quad \big(k1\,k2\,m + (2\,k1\,k2 + (k1 + k2)\,m)\,n + (k1 + k2 + m)\,n^2\big)^4\Big)
\end{aligned}
$$

Appendix C

Some useful calculations: variances.

$$\begin{aligned}
\operatorname{var}(w_{i1}^{XC}+w_{i2}^{XC}) &= \left[\left(B_{i1}^{XC}\right)^2+\left(\beta_{i2}^{XC}\right)^2\right]\operatorname{var}x + 2B_{i1}^{XC}\beta_{i2}^{XC}\operatorname{cov}(x_1,x_2)\\
&= \left(B_i^{XC}\right)^2(k_1+k_2+m)\sigma^2\\
\operatorname{var}(w_{i1}^{YC}+w_{i2}^{YC}) &= \left[\left(G_{i1}^{YC}\right)^2+\left(\gamma_{i2}^{YC}\right)^2\right]\operatorname{var}y_i + \left[\left(D_{i1}^{YC}\right)^2+\left(\delta_{i2}^{YC}\right)^2\right]\operatorname{var}y_j\\
&\quad + 2G_{i1}^{YC}\gamma_{i2}^{YC}\operatorname{cov}(y_{i1},y_{i2}) + 2D_{i1}^{YC}\delta_{i2}^{YC}\operatorname{cov}(y_{j1},y_{j2})\\
&= \left[\left(G_i^{YC}\right)^2+\left(D_i^{YC}\right)^2\right]n\sigma^2 + \left(G_i^{YC}\right)^2k_i\sigma^2 + \left(D_i^{YC}\right)^2k_j\sigma^2\\
\operatorname{var}(w_{i1}^{XYC}+w_{i2}^{XYC}) &= \left[\left(G_{i1}^{XYC}\right)^2+\left(\gamma_{i2}^{XYC}\right)^2\right](k_i+n)\sigma^2 + \left[\left(D_{i1}^{XYC}\right)^2+\left(\delta_{i2}^{XYC}\right)^2\right](k_j+n)\sigma^2\\
&\quad + \left(B_{i1}^{XYC}\right)^2(k_1+k_2+m)\sigma^2 + \left(2G_{i1}^{XYC}\gamma_{i2}^{XYC}+2G_{i1}^{XYC}B_{i1}^{XYC}+2\gamma_{i2}^{XYC}B_{i1}^{XYC}\right)k_i\sigma^2\\
&\quad + \left(2D_{i1}^{XYC}B_{i1}^{XYC}+2D_{i1}^{XYC}\delta_{i2}^{XYC}+2\delta_{i2}^{XYC}B_{i1}^{XYC}\right)k_j\sigma^2\\
&= \left(G_i^{XYC}\right)^2(k_i+n)\sigma^2 + \left(D_i^{XYC}\right)^2(k_j+n)\sigma^2 + \left(B_i^{XYC}\right)^2(k_1+k_2+m)\sigma^2\\
&\quad + 2G_i^{XYC}B_i^{XYC}k_i\sigma^2 + 2D_i^{XYC}B_i^{XYC}k_j\sigma^2
\end{aligned}$$

Proof of Lemma 4.1.

$$\frac{\partial ACE_i^{XC}}{\partial e_i^{XC}} = \beta_i^{XC} + \frac{k_i}{k_1+k_2+m} - e_i^{XC} = 0$$

$$\frac{\partial ACE_i^{XC}}{\partial \tau_i^{XC}} = \left(\frac{s}{2}\right)\left(\frac{\tau_j^{XC}}{\tau_i^{XC}}\right)^{1/2}\left(\beta_i^{XC} + \frac{k_i}{k_1+k_2+m}\right) - \tau_i^{XC} = 0$$

Solving simultaneously for τ_i and rearranging terms yields Lemma 4.1. ▲

Proof of Lemma 4.2.

$$\Pi_1^{XC} = B_1^{XC} + B_2^{XC} + \frac{s^2}{2}\left(B_1^{XC}B_2^{XC}\right)^{1/2} - \frac{\left(B_1^{XC}\right)^2}{2} - \frac{\left(B_2^{XC}\right)^2}{2} - \frac{\left(B_1^{XC}\right)^{3/2}\left(B_2^{XC}\right)^{1/2}}{2}$$

$$- \frac{\left(B_2^{XC}\right)^{3/2}\left(B_1^{XC}\right)^{1/2}}{2} - \frac{r}{2}\sigma^2\left(B_i^{XC}\right)^2(k_1 + k_2 + m) + 2(\mu_1 + \mu_2)$$

$$\frac{\partial \Pi_1^{XC}}{\partial B_i^{XC}} = 1 - B_i^{XC}\left(1 + r\sigma^2(k_1 + k_2 + m)\right) + \frac{s^2}{16}\left(\frac{B_j^{XC}}{B_i^{XC}}\right)^{1/2}\left(4 - B_j^{XC} - 3B_i^{XC}\right)$$

Imposing symmetry yields:

$$1 - B_i^{XC}\left(1 + r\sigma^2(2k + m)\right) + \frac{s^2}{4}\left(1 - B_i^{XC}\right) = 0$$

$$1 + \frac{s^2}{4} = B_i^{XC}\left(1 + r\sigma^2(2k + m) + \frac{s^2}{4}\right)$$

Total incentive $\Rightarrow B_i^{XC} = \dfrac{s^2 + 4}{s^2 + 4 + 4r\sigma^2(2k + m)}$

Explicit incentive $\Rightarrow \beta_i^{XC} = \dfrac{s^2 + 4}{s^2 + 4 + 4r\sigma^2(2k + m)} - \dfrac{k}{2k + m}$ ▲

Proof of Propositions 4.1, 4.2, 4.3 and Corollary 4.1.

All propositions and the corollary follow directly from the derivations and discussion in the text.

Proof of Lemma 4.3.

$$\frac{\partial ACE_i^{YC}}{\partial e_i^{YC}} = \gamma_i^{YC} + \frac{k_i}{k_i + n} - e_i^{YC} = 0, \quad \frac{\partial ACE_i^{YC}}{\partial \tau_i^{YC}} = f_{ii}\left(\gamma_i^{YC} + \frac{k_i}{k_i + n}\right) + f_{ji}\delta_i^{YC} - \tau_i^{YC} = 0 \blacksquare$$

Proof of Lemma 4.4.

Let $G_{i1}^{YC} \equiv \gamma_{i1}^{YC} + \dfrac{k_i}{k_i + n}$ and $D_{i1}^{YC} \equiv \delta_{i1}^{YC}$.

Then $\Pi_1^{YC} = G_1^{YC} + G_2^{YC} - \frac{\left(G_1^{YC}\right)^2}{2} - \frac{\left(G_2^{YC}\right)^2}{2}$

$$- \frac{\left(f_{11}\left(G_1^{YC} - \frac{k_1}{k_1 + n} \right) + f_{21} D_1^{YC} \right)^2}{2} - \frac{\left(f_{22}\left(G_2^{YC} - \frac{k_2}{k_2 + n} \right) + f_{12} D_2^{YC} \right)^2}{2}$$

$$+ s\left(f_{11}\left(G_1^{YC} - \frac{k_1}{k_1 + n} \right) + f_{21} D_1^{YC} \right)^{1/2} \left(f_{22}\left(G_2^{YC} - \frac{k_2}{k_2 + n} \right) + f_{12} D_2^{YC} \right)^{1/2}$$

$$- \frac{r}{2}\sigma^2 \left[\left(G_i^{YC}\right)^2 (k_i + n) + \left(D_i^{YC}\right)^2 (k_j + n) \right] + 2(\mu_1 + \mu_2)$$

Solve for G_i^{YC} : $\frac{\partial}{\partial G_i^{YC}} = 1 + \frac{s}{2} f_{ii} \frac{\left(f_{jj}\left(G_j^{YC} - \frac{k_j}{k_j + n} \right) + f_{ij} D_j^{YC} \right)^{1/2}}{\left(f_{ii}\left(G_i^{YC} - \frac{k_i}{k_i + n} \right) + f_{ji} D_i^{YC} \right)^{1/2}}$

$$- G_i^{YC} (1 + f_{ii}{}^2 + r\sigma^2 (k_i + n)) + \frac{f_{ii} k_i}{k_i + n} - f_{ii} f_{ji} D_i^{YC} = 0$$

Imposing symmetry yields:

$$1 + \frac{s}{2} f_{ii} + \frac{f_{ii} k}{k + n} - f_{ii} f_{ji} D_i^{YC} - G_i^{YC} (1 + f_{ii}{}^2 + r\sigma^2 (k + n)) = 0$$

Total incentive $\Rightarrow G_i^{YC} = \frac{1 + \frac{s}{2} f_{ii} + \frac{f_{ii} k}{k + n} - f_{ii} f_{ji} D_i^{YC}}{1 + f_{ii}{}^2 + r\sigma^2 (k + n)}$

$$= \frac{1}{D_Y} \left[f_{ji}{}^2 + r\sigma^2 (k + n) \left(1 + \frac{s}{2} f_{ii} \right) \right]$$

where $D_Y \equiv f_{ji}{}^2 + r\sigma^2 (k + n) \left[1 + f_{ii}{}^2 + f_{ji}{}^2 + r\sigma^2 (k + n) \right]$

Explicit incentive $\Rightarrow \gamma_i^{YC} = \frac{1}{D_Y} \left[f_{ji}{}^2 + r\sigma^2 (k + n) \left(1 + \frac{s}{2} f_{ii} \right) \right] - \frac{k}{k + n}$

Solve for D_i^{YC} : $\dfrac{\partial}{\partial D_i^{YC}} = \dfrac{s}{2} f_{ji} \dfrac{\left(f_{jj}\left(G_j^{YC} - \dfrac{k_j}{k_j + n} \right) + f_{ij} D_j^{YC} \right)^{1/2}}{\left(f_{ii}\left(G_i^{YC} - \dfrac{k_i}{k_i + n} \right) + f_{ji} D_i^{YC} \right)^{1/2}} - f_{ii} f_{ji} \left(G_i^{YC} - \dfrac{k_i}{k_i + n} \right)$

$$- f_{ji}{}^2 D_i^{YC} - D_i^{YC} r\sigma^2 (k_j + n)) = 0$$

Imposing symmetry yields:

$$\frac{s}{2} f_{ji} - f_{ii} f_{ji} \left(G_i^{YC} - \frac{k}{k+n} \right) - D_i^{YC} \left(f_{ji}{}^2 + r\sigma^2 (k+n) \right) = 0$$

Total incentive $\Rightarrow D_i^{YC} = \delta_i^{YC} = \dfrac{\dfrac{s}{2} f_{ji} - f_{ii} f_{ji} \left(G_i^{YC} - \dfrac{k}{k+n} \right)}{f_{ji}{}^2 + r\sigma^2 (k+n)}$

$$= \frac{1}{D_Y} \left[\frac{s}{2} f_{ji} \left(1 + r\sigma^2 (k+n) \right) - f_{ii} f_{ji} \right] \blacktriangle$$

Proofs of Lemmas 4.5 and 4.6: these proofs directly parallel the proofs of Lemma 4.3 and Lemma 4.4, respectively.

Lengthy expressions omitted from the text:

$$\frac{\partial e_i^{XYC}}{\partial k} < 0$$

$$-\frac{r^2\sigma^4}{D_{XY}} \left[\begin{array}{l} s^4 (k+n)^2 \\ +s^2 r\sigma^2 \left(8k^2 m + 16kn(k+m) + 8n^2(3k+m) + 12n^3 + s^2 n(k+n)^2 \right) \\ +4r^2\sigma^4 \left(\begin{array}{l} 8kn(m+n)(m+2n) + 4k^2(m+2n)^2 \\ +4n^2(m^2 + 2mn + 2n^2) + s^2 n(k+n)(m+2n) \end{array} \right) \end{array} \right]$$

$$\frac{\partial \tau_i^{XYC}}{\partial k} < 0$$

$$-\frac{2snr\sigma^2}{D_{XY}} \left[s^2 n + r\sigma^2 \left(4n^2 + s^2 (k+n)(k+3n) \right) + 2r^2\sigma^4 (k+n)^2 (2m + n(s^2+4)) \right]$$

$$
\begin{aligned}
&\frac{\partial^2 e_i^{XYC}}{\partial k^2} > 0 \\
&\quad \Big(2 r^2 \sigma^4 \\
&\quad \Big(4 n^3 s^4 + (k+n)^3 s^6 + \\
&\qquad r s^2 \big(16 n^3 (m+3n) + 12 (k+n) \big(k^2 m + 2 k (k+m) n + (3k+m) n^2 + 2 n^3\big) s^2 + n (k+n)^3 s^4\big) \sigma^2 + \\
&\qquad 8 r^2 \\
&\qquad\; \big(8 n^4 (m+2n) + \\
&\qquad\qquad 2 \big(3 k^3 (m+2n)^2 + 3 k^2 n (m+2n)(3m+4n) + 3 k n^2 \big(3m^2 + 9mn + 8n^2\big) + \\
&\qquad\qquad\quad n^3 \big(3m^2 + 9mn + 10n^2\big)\big) s^2 + n (k+n)^3 (m+2n) s^4\big) \sigma^4 + \\
&\qquad 16 r^3 \big(4 (n(m+n) + k(m+2n)) \big(2kn(m+n)(m+2n) + k^2 (m+2n)^2 + n^2 \big(m^2 + 2mn + 4n^2\big)\big) + \\
&\qquad\qquad n (k+n)^3 (m+2n)^2 s^2\big) \sigma^6\Big)\Big) / D_{XY}{}^3
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2 \tau_i^{XYC}}{\partial k^2} > 0 \\
&\quad \Big(4 n r s \sigma^2 \\
&\quad \Big(n s^4 + n r s^2 \big(4m + 12n + 3(k+n) s^2\big) \sigma^2 + \\
&\qquad r^2 \big(16 n^2 (m+2n) + 4n(3k(m+3n) + n(3m+8n)) s^2 + (k+n)^2 (k+4n) s^4\big) \sigma^4 + \\
&\qquad 2 r^3 \big(8 n^2 (3k(m+2n) + n(3m+4n)) + 2 (k+n)^2 (m+2n)(2k+5n) s^2 + n (k+n)^3 s^4\big) \sigma^6 + \\
&\qquad 8 (k+n)^3 (m+2n) r^4 \big(2m + n\big(4 + s^2\big)\big) \sigma^8\Big)\Big) / D_{XY}{}^3
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2 e_i^{XYC}}{\partial s \partial k} = \\
&\quad \Big(8 n r^2 s \sigma^4 \big(n^2 (k+n) s^2 - r \big(4 n^2 (km + (m-n) n) + (k+n)^2 (km + (m-3n) n) s^2\big) \sigma^2 - \\
&\qquad (k+n) r^2 \big(4 k^2 m^2 + 8 k m (k+m) n + 4 m (4k+m) n^2 + 8 m n^3 - 8 n^4 + \\
&\qquad\qquad (k+n)^2 ((m-2n) n + k(m+2n)) s^2\big) \sigma^4 - 4 (k+n)^3 (m+2n)(mn + k(m+2n)) r^3 \sigma^6\big)\Big) / \\
&\qquad\qquad D_{XY}{}^3
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2 \tau_i^{XYC}}{\partial s \partial k} = \\
&\quad \Big(2 n r \sigma^2 \big(n (k+n) s^4 + r s^2 \big(-12 n (mn + k(m+n)) + (k+n)^2 (k+4n) s^2\big) \sigma^2 + \\
&\quad r^2 \big(-16 n^2 (n(m+n) + k(m+2n)) - 12 n (k+n)(3km + 4kn + 3mn) s^2 + (k+n)^3 (k+5n) s^4\big) \sigma^4 + \\
&\quad 2 (k+n) r^3 \big(8 \big(-k^2 (m+2n)^2 - 2kn(m+2n)^2 - n^2 \big(m^2 + 4mn + 2n^2\big)\big) - \\
&\qquad 12 n (k+n)(2km + 3kn + 2mn) s^2 + n (k+n)^3 s^4\big) \sigma^6 - \\
&\quad 8 (k+n)^3 (mn + k(m+2n)) r^4 \big(2m + n\big(4 + 3 s^2\big)\big) \sigma^8\big)\Big) / D_{XY}{}^3
\end{aligned}
$$

$$\frac{\partial \Pi_1^{XYC}}{\partial s} > 0$$

$$\Big((k+n)\, s\left(4nr\sigma^2 + s^2\left(1 + kr\sigma^2 + nr\sigma^2\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 8(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(2m+n+2mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m\left(2+4nr\sigma^2\right) + n\left(3+4nr\sigma^2\right)\right)\right)\right)\Big) \Big/$$
$$\Big(2$$
$$\left(k^2 r\sigma^2\left(s^2 + 4(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(m+n+mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m+2mnr\sigma^2 + 2n\left(1+nr\sigma^2\right)\right)\right)\right)^2\Big)$$

$$\frac{\partial^2 \Pi_1^{XYC}}{\partial s^2} > 0$$

$$\Big((k+n)$$
$$\Big(2(k+n)s^2\left(1+kr\sigma^2+nr\sigma^2\right)\left(4nr\sigma^2 + s^2\left(1+kr\sigma^2+nr\sigma^2\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 4(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(m+n+mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m+2mnr\sigma^2 + 2n\left(1+nr\sigma^2\right)\right)\right)\right) -$$
$$4(k+n)s^2\left(1+kr\sigma^2+nr\sigma^2\right)\left(4nr\sigma^2 + s^2\left(1+kr\sigma^2+nr\sigma^2\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 8(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(2m+n+2mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m\left(2+4nr\sigma^2\right) + n\left(3+4nr\sigma^2\right)\right)\right)\right) +$$
$$2s^2\left(1+kr\sigma^2+nr\sigma^2\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 4(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(m+n+mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m+2mnr\sigma^2 + 2n\left(1+nr\sigma^2\right)\right)\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 8(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(2m+n+2mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m\left(2+4nr\sigma^2\right) + n\left(3+4nr\sigma^2\right)\right)\right)\right) +$$
$$\left(4nr\sigma^2 + s^2\left(1+kr\sigma^2+nr\sigma^2\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 4(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(m+n+mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m+2mnr\sigma^2 + 2n\left(1+nr\sigma^2\right)\right)\right)\right)$$
$$\left(k^2 r\sigma^2\left(s^2 + 8(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(2m+n+2mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m\left(2+4nr\sigma^2\right) + n\left(3+4nr\sigma^2\right)\right)\right)\right)\Big)\Big) \Big/$$
$$\Big(2$$
$$\left(k^2 r\sigma^2\left(s^2 + 4(m+2n)r\sigma^2\right) + n\left(s^2\left(1+nr\sigma^2\right) + 4r\sigma^2\left(m+n+mnr\sigma^2\right)\right) +\right.$$
$$\left.k\left(s^2\left(1+2nr\sigma^2\right) + 4r\sigma^2\left(m+2mnr\sigma^2 + 2n\left(1+nr\sigma^2\right)\right)\right)\right)^3\Big)$$

$$\frac{\partial \Pi_1^{XYC}}{\partial k} < 0$$

$$\begin{aligned}
&(r\sigma^2 \\
&\quad (-(k^2 + 2kn + 2n^2)s^4 - 2rs^2(4k^2 m + 8k(k+m)n + 4(3k+m)n^2 + 8n^3 + n(k+n)(k+2n)s^2) \\
&\qquad \sigma^2 - \\
&\qquad 2r^2(8(2kn(m+n)(m+2n) + k^2(m+2n)^2 + n^2(m^2 + 2mn + 2n^2)) + \\
&\qquad\quad 4n(k+n)^2(m+2n)s^2 + n^2(k+n)^2 s^4)\sigma^4)) / D_{XY}{}^2
\end{aligned}$$

$$\frac{\partial^2 \Pi_1^{XYC}}{\partial k^2} < 0$$

$$\begin{aligned}
&(r\sigma^2 \\
&\quad (((k+n)s^2 + r(4(n(m+n) + k(m+2n)) + (k+n)^2 s^2)\sigma^2 + 4(k+n)(mn + k(m+2n))r^2\sigma^4) \\
&\qquad (-2(k+n)s^4 - 2rs^2(n(8m + 3n(4+s^2)) + 2k(4m + n(8+s^2)))\sigma^2 - \\
&\qquad\quad 4r^2(8(m+2n)(n(m+n) + k(m+2n)) + 4n(k+n)(m+2n)s^2 + n^2(k+n)s^4)\sigma^4) - \\
&\quad 2(s^2 + 2r(2m + 4n + (k+n)s^2)\sigma^2 + 8(n(m+n) + k(m+2n))r^2\sigma^4) \\
&\qquad (-(k^2 + 2kn + 2n^2)s^4 - \\
&\qquad\quad 2rs^2(4k^2 m + 8k(k+m)n + 4(3k+m)n^2 + 8n^3 + n(k+n)(k+2n)s^2)\sigma^2 - \\
&\qquad\quad 2r^2(8(2kn(m+n)(m+2n) + k^2(m+2n)^2 + n^2(m^2 + 2mn + 2n^2)) + \\
&\qquad\qquad 4n(k+n)^2(m+2n)s^2 + n^2(k+n)^2 s^4)\sigma^4))) / D_{XY}{}^3
\end{aligned}$$

$$\frac{\partial^2 \Pi_1^{XYC}}{\partial s \partial k} < 0$$

$$\begin{aligned}
&-(16nr^2 s\sigma^4(6k^2 n(m+n)r\sigma^2(1 + 2nr\sigma^2)(2(m+2n)r\sigma^2 + s^2(1 + nr\sigma^2)) + \\
&\qquad k^4(m+2n)r^2\sigma^4(4(m+2n)r\sigma^2 + s^2(1 + 2nr\sigma^2)) + \\
&\qquad mn^2(1 + nr\sigma^2)(4nr\sigma^2(1 + mr\sigma^2 + 2nr\sigma^2) + s^2(1 + 3nr\sigma^2 + 2n^2 r^2\sigma^4)) + \\
&\qquad k^3 r\sigma^2(4m^2 r\sigma^2(1 + 4nr\sigma^2) + m(4nr\sigma^2(3 + 14nr\sigma^2) + s^2(1 + 8nr\sigma^2 + 8n^2 r^2\sigma^4)) + \\
&\qquad\quad n(8nr\sigma^2(1 + 6nr\sigma^2) + s^2(1 + 12nr\sigma^2 + 12n^2 r^2\sigma^4))) + \\
&\qquad kn(4m^2 nr^2\sigma^4(3 + 4nr\sigma^2) + n(1 + 2nr\sigma^2)^2(4nr\sigma^2 + s^2(1 + nr\sigma^2)) + \\
&\qquad\quad m(4nr\sigma^2(1 + 9nr\sigma^2 + 10n^2 r^2\sigma^4) + s^2(1 + 9nr\sigma^2 + 16n^2 r^2\sigma^4 + 8n^3 r^3\sigma^6))))) / \\
&\quad (k^2 r\sigma^2(s^2 + 4(m+2n)r\sigma^2) + n(s^2(1 + nr\sigma^2) + 4r\sigma^2(m + n + mnr\sigma^2)) + \\
&\qquad k(s^2(1 + 2nr\sigma^2) + 4r\sigma^2(m + 2mnr\sigma^2 + 2n(1 + nr\sigma^2))))^3
\end{aligned}$$

Glossary

c_i	Cost of effort for agent *i* (*i*=1,2)
e_i	Individual effort by agent *i* (*i*=1,2)
τ_i	Cooperative effort (teamwork) by agent *i* (*i*=1,2)
p_i	Task expertise for agent *i* (*i*=1,2)
q_i	Team expertise for agent *i* (*i*=1,2)
r_i	Coefficient of risk aversion for agent *i* (*i*=1,2)
s	Level of synergy in team output
w_i	Compensation contract for agent *i* (*i*=1,2)
x	Team output
y_i	Performance measure of agent *i*'s individual effort contribution (*i*=1,2)
z_i	Performance measure of agent *i*'s cooperative effort contribution (*i*=1,2)
α_i	Fixed wage for agent *i* (*i*=1,2)
β_i	Incentive weight for agent *i* (*i*=1,2) on team output, x
γ_i	Incentive weight for *i* (*i*=1,2) on his measure of individual effort, y_i
δ_i	Incentive weight for *i* (*i*=1,2) on agent *j*'s individual effort measure, y_j
κ_i	Incentive weight for *i* (*i*=1,2) on his measure of cooperative effort, z_i
λ_i	Incentive weight for *i* (*i*=1,2) on agent *j*'s cooperative effort measure, z_j
ρ_n	Correlation between performance measures n_i and n_j ($n = y, z$)
σ_n^2	Variance of performance measure n ($n = x, y_i, y_j, z_i, z_j$)
Σ_n^2	Variance of performance measure n, net of correlation

References

Alchian, A. A. and H. Demsetz. 1972. Production, information costs and economic organization. *American Economic Review* 23 (1): 7-30.

Andersson, F. 2002. Career concerns, contracts, and effort distortions. *Journal of Labor Economics* 20 (1): 42-58.

Arya, A., J. Fellingham, and J. Glover. 1997. Teams, repeated tasks, and implicit incentives. *Journal of Accounting & Economics* 23 (1): 7-30.

Auriol, E., G. Friebel, and L. Pechlivanos. 2002. Career concerns in teams. *Journal of Labor Economics* 20 (2): 289-307.

Autrey, R. L., S. S. Dikolli, and D. P. Newman. 2005. Career concerns and the contracting demand for public and private performance measures. *Working Paper, University of Texas at Austin*.

Berck, P. and J. Lipow. 2000. Managerial reputation and the 'endgame'. *Journal of Economic Behavior & Organization* 42 (2): 253-263.

Bernhardt, D. and D. Scoones. 1993. Promotion, turnover, and preemptive wage offers. *American Economic Review* 83 (4): 771.

Borland, J. 1992. Career concerns: Incentives and endogenous learning in labour. *Journal of Economic Surveys* 6 (3): 251.

Carrillo, J. D. 2003. Job assignments as a screening device. *International Journal of Industrial Organization* 21 (6): 881.

Che, Y. K. and S. W. Yoo. 2001. Optimal incentives for teams. *American Economic Review* 91 (3): 525-541.

Clark, D. 2004. *PeopleSoft, Conway set severance*, The Wall Street Journal, October 20, 2004 ed.: B3: Dow Jones & Company, Inc.

Datar, S. M., S. Kulp, and R. A. Lambert. 2001. Balancing performance measures. *Journal of Accounting Research* 39 (1): 75-92.

Dewatripont, M., I. Jewittt, and J. Tirole. 1999. The economics of career concerns, part 1: Comparing information structures. *Review of Economic Studies* 66 (226): 183.

Dikolli, S. S. 2001. Agent employment horizons and contracting demand for forward-looking performance measures. *Journal of Accounting Research* 39 (3): 467-480.

Farber, H. S. and R. Gibbons. 1996. Learning and wage dynamics. *Quarterly Journal of Economics* 111 (4): 1007-1047.

Feltham, G. A. and J. Xie. 1994. Performance measure congruity and diversity in multi-task principal/ agent relations. *The Accounting Review* 69 (3): 429-453.

Gibbons, R. and K. J. Murphy. 1992. Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy* 100 (3): 468-593.

Hemmer, T. 1995. On the interrelation between production technology, job design, and incentives. *Journal of Accounting and Economics* 19: 209-245.

Hoffler, F. and D. Sliwka. 2003. Do new brooms sweep clean? When and why dismissing a manager increases the subordinates' performance. *European Economic Review* 47 (5): 877-890.

Holmstrom, B. 1982. Moral hazard in teams. *Bell Journal of Economics* 13 (2): 324-340.

Holmstrom, B. and J. Ricart i Costa. 1986. Managerial incentives and capital management. *Quarterly Journal of Economics* 101 (4): 835.
Itoh, H. 1993. Coalitions, incentives, and risk sharing. *Journal of Economic Theory* 60: 410-427.
Jeon, S. 1996. Moral hazard and reputational concerns in teams: Implications for organizational choice. *International Journal of Industrial Organization* 14 (3): 297-315.
____. 1998. Reputational concerns and managerial incentives in investment decisions. *European Economic Review* 42 (7): 1203-1219.
Kaarboe, O. M. and T. E. Olsen. 2004. Career concerns, monetary incentives and job design. *Working Paper, University of Bergen*.
Keating, S. A. 1997. Determinants of divisional performance evaluation practices. *Journal of Accounting and Economics* 24: 243-273.
Lambert, R. A. 2001. Contracting theory and accounting. *Journal of Accounting & Economics* 32 (1-3, December): 3-87.
Lawford, G. R. 2003. Beyond success: Achieving synergy in teamwork. *Journal for Quality & Participation* 26 (3): 23-27.
Macho-Stadler, I. and J. D. Perez-Castrillo. 1993. Moral hazard with several agents: The gains from cooperation. *International Journal of Industrial Organization* 11: 73-100.
McAfee, R. P. and J. McMillan. 1991. Optimal contracts for teams. *International Economic Review* 32 (3): 561-577.
Meyer, M. A. and J. Vickers. 1997. Performance comparisons and dynamic incentives. *Journal of Political Economy* 105 (3): 547-581.
Nagar, V. 1999. The role of the manager's human capital in discretionary disclosure. *Journal of Accounting Research* 37: 167-181.
Nagarajan, N. J., K. Sivaramakrishnan, and S. S. Sridhar. 1995. Managerial entrenchment, reputation and corporate investment myopia. *Journal of Accounting, Auditing & Finance* 10 (3): 565-585.
Parker, G., J. McAdams, and D. Zielinski. 2000. *Rewarding teams: Lessons from the trenches* (1st ed.). San Francisco, CA: Jossey-Blass Inc.
Porter, M. E. 1996. What is strategy? *Harvard Business Review* (November/December): 61-78.
Rose, D. C. 2002. Marginal productivity analysis in teams. *Journal of Economic Behavior & Organization* 48: 355-363.
Salas, E., C. S. Burke, and J. A. Cannon-Bowers. 2000. Teamwork: Emerging principles. *International Journal of Management Reviews* 2 (4): 339-356.
Senbongi, S. and J. E. Harrington. 1995. Managerial reputation and the competitiveness of an industry. *International Journal of Industrial Organization* 13 (1): 95-110.
Sridhar, S. S. 1994. Managerial reputation and internal reporting. *Accounting Review* 69 (2): 343-363.
Tadelis, S. 2002. The market for reputations as an incentive mechanism. *Journal of Political Economy* 110 (4): 854-882.
Vanderveen, T. D. 1995. Optimal contracts for teams - a note on the results of McAfee and McMillan. *International Economic Review* 36 (4): 1051-1058.

Watts, R. L. 2003. Conservatism in accounting part 1: Explanations and implications. *Accounting Horizons* 17: 207-221.

Zabojnik, J. 2001. *On the efficiency of markets for managers.*, Economic Theory, Vol. 18: 701: Springer - Verlag New York, Inc.

Zhang, L. 2003. Complementarity, task assignment, and incentives. *Journal of Management Accounting Research* 15: 225-246.

Vita

Romana Louise Autrey was born in Madison, Wisconsin on November 18, 1967, the daughter of Mary Louise Freimanis and Ansis Hagen Freimanis. After completing her work at The Louisiana School for Math, Science and the Arts, Natchitoches, Louisiana, in 1985, she entered California Institute of Technology in Pasadena, California, where she attended the 1985-86 and 1987-88 academic years. She attended Northwestern State University of Louisiana in Natchitoches, Louisiana in the 1986-1987 academic year. She was employed at Automotive Rentals, Inc. as a client service representative from 1989 to 1992. She entered the California State University in Hayward, California (now California State University, East Bay) in January 1990 and received the degree of Bachelor of Science in Business Administration, summa cum laude, from California State University of Hayward in June 1994. Following graduation, she was employed at KPMG LLP in the San Francisco Bay Area. In July 2000 she entered the Red McCombs School of Business at The University of Texas at Austin.

Permanent address: 17 Magnolia Avenue #2, Cambridge, Massachusetts 02138

This dissertation was typed by the author.